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A STANDARD GAUGE SYSTEM.

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IN a paper presented at the May meeting of the Society last year, a statement, or "report of progress" was submitted, showing the method adopted by the Pratt & Whitney Company, of Hartford, Conn., by which the question of practically establishing a standard for size gauges was to be scientifically determined, accurately subdividing the Imperial yard into feet and inches and fractional parts of an inch, and describing briefly the extent to which was carried the scientific research found absolutely necessary for such an undertaking; it remains now to present to the consideration of those who may be interested, a statement of the results proceeding from the practical application of all the thorough, conscientious investigation of Professor Rogers, of Harvard College Observatory, whose invaluable experience and professional services, in obtaining for the company the transfer and subsequent subdivision of the British yard, gave the foundation for what has been accomplished, enabling the company to feel warranted in earnestly inviting an inspection of the means now available for the production of standard sizes, and asking for it the indorsement of the engineering profession, should it be found worthy of such necessary support.

The comparator referred to in the previous paper, has been placed in position upon brick piers in a room outside the main building, erected especially for it, and is comparatively free from the jar and tremor of the machinery, even unaffected by the jar of passing trains, the tracks of the New York, New Haven and Hartford, and of the New York and New England Railroads being quite near, the rigidity of the instruments and the excellent workmanship in its construction preventing any perceptible vibration during an observation, even when the high power microscopes, magnifying 150 diameters, are used.

The illumination required in using these microscopes is perfectly attained by reflection, using a plate-glass mirror placed outside of the window of the comparing room, at an inclination of 45 degrees, giving clear diffused light, cloudy weather even improving the general effect, as the light is then whiter than that reflected from a clear blue sky, and the lines on the standard bar, as seen through the medium of the Tolles' illuminating prisms with which the objectives

are fitted, are clearly and sharply defined at any time during daylight, and in any position of the microscope plate; by thus avoiding the use of artificial light and consequent effect of a variable temperature, far more satisfactory results are obtained.

The investigation for the determination of the necessary corrections for errors due to horizontal or vertical curvature of the path of the microscope plate, has shown conclusively the unexcelled workmanship in the construction of the comparator; as an instance of how slight these errors really are, it was found after repeated observations the means being carefully collected, that the horizontal curvature, *i. e.*, the bending sidewise of the cylindrical guides or ways upon which the microscope plate slides, at the part investigated, was that having a radius of *eleven miles*, and a consequent correction to be applied of about one ten-thousandth of an inch in eighteen inches, the latter distance referring to the position of the measured standard when placed either side of the line of motion of the center of the microscope plate, moving between fixed stops, and which is the constant quantity to which is referred the subdivisions of the standard bar; as the microscope is usually within one inch, rarely over two and a half, from the center line of the stops or caliper jaws between which the end measure pieces or the cylindrical gauges are placed, this correction evidently becomes too small to be applied practically, within the limit of a six-inch gauge; for a foot or a yard it would become necessary, as the variation of the chords of the subtended arcs then becomes quite perceptible. The errors due to inequality of temperature in the standard steel bar and the hardened steel end measure gauges must be carefully guarded against, the latter effect being far more important, practically, and often very misleading. In the case of a four-inch hardened steel end measure gauge experimented upon, the coefficient of expansion being nearly one one-hundred thousandth of its length,* one degree of change of temperature from that maintained in the reference bar, introduces an error of nearly one twenty-five thousandth of an inch in the total length, and hence, as a change or inequality of five or even ten degrees might be easily overlooked, the four-inch gauge would be found to be from one five thousandth ($\frac{1}{5000}$) to one twenty-five hundredth ($\frac{1}{2500}$) of an inch too short, when equality of temperature is restored; when it is asserted that an actual variation of so minute a quantity as *one thirty-thousandth* of an inch, and even less, can be readily detected by any tool-maker familiar with the use of an ordinary micrometer or a close gauge, the importance of keeping *within* this limit is apparent,—of course, this shortening effect is not so marked in smaller sizes, still the ratio is the same, and this error must be carefully avoided.

The subdivisions upon the six-inch hardened steel standard bar have been carefully investigated upon the new comparator, to determine how nearly these inch spaces equal each other, the total length of the four inches which are ruled upon this six-inch bar being *exactly standard* at 62 degrees Fahrenheit, according to the official report of Professor Rogers, received December 1st, 1881, and in this report, the results obtained by him, determining this relation of the inch spaces to each other, were found to agree closely with the results obtained by me, as the following comparison will show, these minute corrections being necessary in accurately determining the subdivision of the Imperial yard, which they represent:

Corrections.	Prof. Roger's Report.	Results obtained.	
Total 1 inch, add....	0.000008	0.000008	($\frac{1}{3}\frac{1}{6}$ yd.)
Total 2 ins., *subtract.*	0.000026	0.000027	($\frac{1}{1}\frac{1}{3}$ ")
Total 3 ins., *subtract.*	0.000005	0.000006	($\frac{1}{1}\frac{1}{2}$ ")
Total 4 inches.....	correct.	correct.	($\frac{1}{9}$ ")

(The errors being counted from the first line.)

The results given in the last column are the *means* of a number of observations, taken at different times and under various conditions of temperature, &c., and cover a period of about four weeks, the final results having been obtained December 31st.

The value of the divisions of the micrometer employed was carefully determined in order to reduce them to the same unit used by Professor Rogers, and was found, using the microscope marked

* 0.0000066+

"B," to equal $\frac{1}{25000}$ of an inch (0.000016 nearly).

When it is considered that the two results were obtained under different conditions, using different microscopes, and with comparators differing in construction, the correctness of the *principle* upon which the comparison is founded, certainly needs no other proof.

The method of obtaining this relation of the separate inches upon the six-inch standard, was referred to in the former paper, and is that of comparing each inch with a constant distance moved over by the microscope plate between fixed stops, a constant pressure of contact being obtained by the use of electro-magnets, the separate inch spaces being thus referred to an *invariable* quantity or distance, and their relation to each other consequently determined.

To explain more fully this operation, the method adopted is as follows: A series of readings are taken at the zero or initial line of the first inch space, using the micrometer referred to, the microscope plate being held firmly against the fixed stop by the electro-magnet; the microscope then moves with the sliding plate until the latter is in contact with the other fixed stop, and held by the electro-magnet, the plate having moved as nearly an inch as it may conveniently be done,—generally a little more than an inch, in order to have the *sign* of the reading always the same,—from three to five readings of the micrometer are then taken at each position of the microscope, and the order reversed, to eliminate possible error; the first inch is thus compared with a fixed quantity, and the same operation repeated for the remaining inch spaces.

The difference between the distance moved by the microscope plate and the distance between the defining lines representing inches, is found by subtracting the *means* of the readings obtained, and thus eliminating the possible error of any single observation.

The following is a series of micrometer readings, and comprises the means of all the observations by which the corrections of the separate inches were obtained, illustrating the system adopted, and which has been used invariably by Professor Rogers in his investigations of the subdivisions of the yard and meter bars, now in the possession of the Pratt & Whitney Company.

COMPARISON OF INCHES.

	L.	R.	L.	R.	
	58.1	65.8	68.0	55.5	
1st inch.....	58.6	66.0	58.1	65.8	Reverse order.
	58.5	65.4	58.0	65.8	
Mean.....	58.4	65.7	58.0	65.7	
	58.0	65.7			
Mean.....	58.2	65.7			

R−L=+7.5 divisions of micrometer.

	L.	R.	L.	R.	
	56.2	65.8	55.4	65.4	
2d inch.....	55.6	65.8	56.0	65.4	Reverse order.
	56.2	66.0	55.4	66.0	
Mean.....	56.0	65.8	55.6	65.6	
	55.6	65.6			
Mean.....	55.8	65.7			

R−L=+9.9.

	L.	R.	L.	R.
	55.0	63.0	57.0	63.0
3d inch.....	55.8	63.0	57.0	63.5
	55.7	63.6	57.0	62.8
Mean.....	55.5	63.2	57.0	63.1
	57.0	63.1		
Mean.....	56.3	63.1		

R−L=+6.8.

	L.	R.	L.	R.
	57.4	64.3	56.0	64.2
4th inch.....	57.8	64.8	56.1	64.5
	57.8	64.8	56.8	64.5
Mean.....	57.6	64.6	56.3	64.4
	56.3	64.4		
Mean.....	56.9	64.5		

R−L=+7.6

CORRECTION Σ

+7.5+0.45+0.45=correction for 1st in.
+9.9−1.95−1.50=correction for 1st 2 ins.
+6.8+1.15−0.35=correction for 1st 3 "
+7.6+0.35±0.00=correction for 4 total "

Mean+7.95

The differences of the observed inch spaces with respect to the constant quantity obtained by the motion of the microscope plates are added and their means taken, from which the corrections are determined with the proper signs; these corrections for the separate inches are added, the final result being evidently zero, the column under "Σ" showing this algebraic sum and the total error reckoned from the first line.

It may be appropriate here to state that the six-inch bar upon which these four-inch spaces are traced, is a standard which is, without doubt, the only hardened steel *line-measure* bar in existence, which is exactly one-ninth part of the Imperial yard at 62 degrees Fahrenheit, and Professor Rogers guarantees it as such in his report, before referred to.

In order to apply these subdivisions, which include all sizes, from one-sixteenth of an inch to four inches, varying by sixteenths, to a practical form, fixtures have been provided and are in constant use, for reducing to *end* measure the distances thus accurately spaced, a caliper attachment has also lately been added, from plans proposed by Professor Rogers, by which the diameter of existing cylindrical gauges, as well as the length of end-measure pieces from one-sixteenth of an inch to six inches may be tested with the same precision that characterizes the investigation of the linear spacing of the standard bar, also providing means for a rigid inspection of finished gauges before they leave the works, thereby insuring uniformity.

To illustrate how nearly alike two pieces may be made, two standard inch end-measure gauges were worked down under the microscope, independently of each other, using the lines upon the ruled standard reference bar and the fixture referred to, which, when compared with each other by the most careful tests, using close or "snap" gauges, and tested thus by tool-makers experienced in work requiring the utmost practical precision, neither piece could be singled out as the larger, the effect of unequal expansion caused by temperature being avoided during the test. Under the microscope, both pieces were found to be exactly alike by a single observation, while the comparison by a series of readings of the two pieces showed a mean difference of one-tenth of a division of the micrometer, and when it is remembered that one division has a value of only $\frac{1}{88500}$th of an inch, the duplication, it may be assured, is certainly satisfactory, and is clearly within a practical, if not a theoretical limit of accuracy.

Having thus the means for closely reproducing established sizes, and no possible wear occurring to the bar from which these sizes are taken, the accurate duplication becomes a comparatively simple operation.

In order to produce standard work within the limit of a yard or a meter, there has been furnished by Professor Rogers, two steel yard and meter bars, referred to in the previous paper, one of these bars tempered, the other being left soft, but having hardened steel plugs, which are adjustable, for the purpose of bringing the surfaces into focus under the microscope; upon the hardened plugs the lines are ruled, both bars having line and end measure, thus providing means for testing the accuracy of the pitch of screw threads of any desired length, and for standard length gauges up to thirty-six inches, or to a meter.

The coefficient of expansion has been determined for each by Professor Rogers with great care, and also the relation, at 62° (F.), between these steel bars and the two bronze standards, both of which are line measure, the latter are described in the previous paper referred to, so that gauges of any size may be made almost independently of temperature, other than the care required in keeping this condition as nearly uniform as possible for both the reference and the measured bar during the time of the transfer, or the determination after having been transferred.

The subject of accurately producing standard leading screws is receiving its share of attention, and those who use micrometers will readily understand its bearing upon the precision with which they may be made, and how unsatisfactory because of the necessary corrections to be applied in many instances, even when standard at some one part of the divided head—a uniform lead or pitch of the screw, however much may be the total error (within a reasonable limit), making a great improvement in their construction.

Besides a complete set of end-measure gauges varying by sixteenths from one-quarter to four inches, there is now ready for inspection a complete plant, consisting of tools and fixtures for producing the standard United States or Franklin Institute thread gauges, every detail having been carefully considered, and every difficulty overcome in the operation for perfecting, not only these standard gauges as to size, but the pitch of the thread, the correct angle, and the width of flat at the

top and bottom of the thread, the accuracy with which these details have been carried out, is now open to the inspection of all who may be interested, and rapid duplication by *machined* work is now an assured success.

It is with the confidence that the "bottom" has finally been reached, that warrants the Pratt and Whitney Company in thus inviting a thorough inspection of the means available, and the methods employed, in producing standard gauges, and earnestly desiring an impartial verdict as to its accuracy and practicability, whether or not, the system as adopted and carried out, has any real merit upon which the confidence of those using gauges for interchangeable work may be safely based.

Discussion.

Professor Robinson: This paper is of the highest merit and of the greatest possible interest to members of the Society, and it is a matter of great satisfaction to the mechanical engineers of this country that there is some one who is able to take hold of this question and treat it so ably as it has been treated; and inasmuch as the methods and results, as stated by the reader, are laid open for inspection by others, I think it is due, in consideration of the amount of effort and expense which has been given to this matter, that the Society, as a matter of duty, should appoint a committee to avail itself of the privileges offered of investigating this, and of making such a statement regarding it as will be found advisable; and undoubtedly this is a thing which the Society will be willing and anxious to indorse in every particular as a standard of the country. For one, I should be glad to see a committee appointed to give this method the attention it deserves.

The President: I suppose the members would be interested if Mr. Pratt would tell us how they went to work in this undertaking. Mr. Pratt told us the story at Hartford, but many gentlemen are here to-day who were not with us then, and I presume it would be very interesting to all to know how Messrs. Pratt and Whitney were led into this prolonged, expensive, and nice investigation.

Mr. Pratt: I will relate the story in the briefest possible way. We were called upon to furnish a set of standard thread gauges, and of course the first thing to do was to get the sizes, and upon examining the different makes of gauges we found no two sets alike, and we were forced to commence, as we thought, at the bottom, and at that time, in sending about a foot-piece that we had obtained, we found that those investigating it did not agree upon its value. Among others Professor Rogers was applied to, to investigate the foot-piece, and he had quite a struggle over it with some of our prominent manufacturers of gauges. They could not agree, and Professor Rogers took it upon himself, at his own expense, to go to Europe, and go to the bottom of the thing. He visited the best authorities in Europe, and spent four months there in investigation. After we had obtained the services of one of the graduates of the Stevens Institute, and in connection with Professor Rogers, we constructed two comparators, one of which Professor Rogers has himself; and one of which we have; they being exactly alike. After Professor Rogers returned he investigated the foot-piece, and found it to be about what he had found it to be before, and then, after the new comparator was finished, he found his statement verified. Previous to this time, fortunately, Professor Rogers had been for several years constructing a ruling machine, and he had it completed about this time. We have gone very carefully into this thing. I do not feel egotistic about it at all. What we want is that every one who is interested in the matter, every society that takes any interest in it, should come and examine our methods and our measurements. If they are good, let us have a standard. If they are not, let us throw them away. It has cost us probably twenty thousand dollars to-day, and I am willing to throw it away if anybody can show us better. We want a standard, and I will not stand in the way of any one else who has a better machine. I feel very much interested in the subject myself; and I think we shall succeed in what we have undertaken.

Professor Egleston: I do not know whether the members of this Society are familiar with the fact that in 1875 the Institute of Mining Engineers appointed a committee on standard gauges, and that committee came to just exactly the

conclusion which is expressed here, that diameters or linear measures ranging in fractions of a millimeter or an inch were the only gauges which were standard or could be standard. It is a peculiar gratification, since a large part of the work fell to my hands, to see that idea so fully sustained. It is a matter of a great deal more importance than perhaps would appear just now. I have heard recently that the English engineers are again agitating the question of standards, and are going back to the old caliper idea. I think this is really a retrograde movement. Most of you may know the fact, but we discovered in the course of our investigations, that in a dozen standard gauges, so-called, of the fixed patterns out of any package no two of them will be alike.

THE CONDITION IN WHICH CARBON EXISTS IN STEEL.

By Prof. F. A. ABEL, C. B., F. R. S., Hon. M. Inst. C. E.

From "Iron."

In a report presented to the Committee on Steel in October, 1881, an account was given of the results of some preliminary experiments, which were carried out with the object of ascertaining, in the first instance, whether any characteristic differences could be established in structure or chemical condition between thin discs of steel cut from one and the same piece of that metal, but differing from each other in regard to the treatment to which they had afterwards been subjected. It will be remembered that it was not possible to throw any light upon the mechanical condition, or structure, of the different specimens, by submitting them to the operation of the solvent (a chromic acid solution) specially selected on account of its gradual action; it being impracticable to check the action at any period when the portions of the discs least acted upon, or not at all attacked, could be retained upon the support on which they were placed, in the positions which they originally occupied in the very thin sheet metal. Considerable differences were found to exist between the total amounts of carbon contained in different discs, from one and the same piece of steel, but in the hardened, tempered, and annealed states respectively. The proportion in the specimens of annealed steel was comparatively very low; and this difference being confirmed by the examination of another series of dics, an inquiry into the course pursued in annealing the steel discs led to further experiments, which appeared clearly to establish the fact that the reduction in the proportion of carbon in the steel during annealing was due to the prolonged exposure of the discs to heat in contact with, or close proximity to, the wrought-iron plates between which they were confined. A thorough confirmation of the correctness of this conclusion being considered desirable by the committee. Mr. Paget was so good as to include, when preparing another series of steel discs from one and the same lot of steel, a number of specimens which were submitted to the annealing process in various ways. In one series, the discs were inclosed in sets of seven, one set between wrought-iron plates, planed and cleaned, and the other between cast-iron plates, planed and cleaned; this combination being again inclosed in wrought-iron and in cast-iron boxes respectively, and packed round with burnt soot. A set of three discs was similarly annealed between black wrought-iron plates, and another set of three between two blocks of fire-clay, inclosed in a cast-iron box and packed round with calcined magnesia. The examination of these sets of discs thus annealed was expected to demonstrate the nature and the extent of the effects of prolonged heating between wrought-iron and cast-iron plates, as to the abstraction of carbon from, or addition of carbon to, thin steel discs in contact with the plates, or separated from them by intervening discs; and also to show what effects, in regard to the condition of carbon in the steel, may be ascribable purely to the process of annealing.

As yet it has been impossible to pro-

ceed far with the examination of these plates, but the extreme decarbonizing effect of prolonged heating of steel in contact with wrought-iron was demonstrated by the following experiment. Some steel discs of the usual dimensions (2.5 inches in diameter and 0.01 inch thick), and containing about 1 per cent. of carbon, were annealed singly between two wrought-iron plates in the manner already described. That is, the arrangement containing the packed plates was raised in an annealing furnace to a bright red heat, sufficient to scale the cast-iron box, but not sufficient to fuse it; the fire was then slackened off, banked up, and the box left in the furnace undisturbed for twenty-four hours. Upon afterwards heating the plates thus treated to redness, and plunging them into cold water, they remained as soft as malleable iron; and the examination of one of the discs by Mr. W. H. Deering (who has carried out the whole of the experimental work connected with this report) showed that the carbon had been reduced to 0.1 per cent. Before proceeding with the comparative examination of the various series of discs, annealed, hardened, and tempered blue and straw, with which the author has been furnished by Mr. Paget, it was considered important to acquire further information regarding the composition and character of the carbon-iron compound which had been obtained, in the experiments described in the last report, by treatment of the thin sheet steel with chromic acid solution (produced by mixing a solution of potassium bichromate, saturated in the cold, with one-twentieth of its volume of pure concentrated sulphuric acid). It was stated in the former report that the cold-rolled and annealed discs thus treated had yielded in different proportions a black scaly or spangly substance, which was attracted by the magnet, and which was found to contain, in combination with iron, an amount of carbon equally practically to the whole amount which had been found to exist in corresponding discs, in the same cold-rolled and annealed condition, taken from the same piece of metal. On the other hand, a disc of hardened steel, which was submitted to the same treatment, yielded only a small quantity of dark particles of similar appearance, in admixture with some lighter colored sediment, and the carbon in the residue, obtained in this instance, amounted only to about one-sixth of the total carbon in this steel. An examination of the proportion which the carbon bore to the iron in this particular residue showed it to be decidedly higher than in the spangly residues furnished by the cold-rolled and annealed samples. The several residues of the latter class resembled each other very closely in composition, and the ratios of the carbon to the iron in each case corresponded closely to that of the carbon in an iron carbide having the formula $F_6 C_1$. In a second experiment made with one of the cold-rolled discs, the metal was exposed to the prolonged action of a solution consisting of the same kind of chromic acid liquor as used in the previous experiments, but mixed with an additional quantity of concentrated sulphuric acid (40 grammes to 500 cubic centimeters of the solution). The heavy grey-black powder which had been separated from this disc (the solution being completed in twenty-four hours) was allowed to remain in the solvent for nine days. Its analysis showed it to contain a comparatively small proportion of iron, which appeared to indicate that the carbon-iron compound, which is at first separated, does not resist the further action of the chromic liquor, in the presence of a considerable excess of sulphuric acid.

The quantities of steel operated upon in these experiments were unavoidably small; and it appeared interesting, and possibly important, to ascertain whether the indications furnished by the results referred to, that the condition of combination of carbon with iron, in steel, differs in samples of one and the same metal if they have been submitted to decidedly different treatment, were confirmed by more extended experiments; also to learn more regarding the nature of the magnetic carbon-iron product eliminated by the action of a slowly oxidizing solvent upon annealed (or cold-rolled) steel; *e. g.* (1) whether its composition is independent of the strength within particular limits, of the chromic solution employed for its elimination; (2) whether, within those limits, a constant quantity of the carbide is obtained from 100 parts of one and the same de-

scription of steel; and (3) what proportions of the carbon in this carbide would remain unconverted upon treatment of the latter with hot chlorhydric acid. With these objects in view the author obtained from Mr. Paget a thin sheet of steel in the same condition as delivered to him from the Birmingham makers, having been cold-rolled and cross cold-rolled, and annealed several times between the various rollings. The weight of this plate was 175 grammes (2,700.6 grains) and its thickness about 0.008 inch (0.2 mm.) Mr. Deering's analysis of a sample of this steel plate, taken from the same part whence specimens were afterwards cut, showed it to contain—carbon, 1.144 per cent.; silicon, 0.166 per cent.; manganese, 0.104 per cent. The individual pieces of the steel plate submitted to the action of the solvent weighed from 7 to about 7.5 grammes (108 to 115.7 grains). Before treatment, the pieces were rubbed bright with emery-flour, then washed with ether, and dried in a clean cloth. The preparation of the chromic acid solutions employed in these experiments is best illustrated by a description of the production of the solvent used in obtaining what will presently be designated preparation 2. This liquor was prepared by adding to a filtered solution of potassium bichromate, saturated in the cold (67°—68° F.) and containing 99 grammes of the salt per 1000 cubic centimeters of solution, a proportion of concentrated sulphuric acid (having the full specific gravity) corresponding to 0.9 gramme of acid per 1 gramme of the potassium salt. The following chemical equation represents the action of a solution of this description upon iron:

$$Fe_2 + K_2Cr_2O_7 + 8H_2SO_4 = Fe_2(SO_4)_3 + Cr_2(SO_4)_3 + 2KHSO_4 + 7H_2O.$$

According to this equation, the theoretical requirement of acid is 0.84867 gramme to 1 gramme of the bichromate, and 1,000 c. c. of the solution thus prepared would suffice to dissolve 9.226 grammes of iron. Each piece of steel immersed in that quantity of liquid weighed, as stated, about 7 grammes (except in the case of preparation 4, when the proportions were about doubled). Therefore the solution employed was always considerably in excess of the amount required to dissolve the metal. (The strength of the solutions was checked by an estimation of the available oxygen contained in them.) The solution used in obtaining preparation 1 was intended to have been prepared from a bichromate solution saturated in the cold, of the precise nature of that just described; its examination showed, however, that it was somewhat weaker, being 0.8 the strength of the solution for preparation 2. Preparation 3 was produced with a much weaker chromic solution; the strength aimed at was 0.5 that of preparation 2, and its actual strength was 0.44. The chromic liquor used for obtaining preparation 4 was prepared by mixing a hot solution of bichromate with the requisite proportion of sulphuric acid (1 of the former to 0.9 of the latter); and the strength aimed at was double that of preparation 2. Two different quantities of the liquid were prepared, but in both cases the strength exceeded that of preparation 2 only by about one-half (being 1.44 of its strength in one case and 1.65 in the other); a little chromic acid having in each case crystallized out, together with the potassium bisulphate, on the cooling of the liquids. The mode of treatment of the steel by the chromic solutions was in all instances alike. The solvent (1000 c. c. in this particular case) was contained in a capacious, somewhat tall glass vessel; and the weighed piece of sheet steel was supported, at about the center of the liquid, upon a diaphragm, or sieve, of platinum wire gauze. Though the surfaces of the steel were perfectly cleaned, as described, it would remain quite unattacked in the liquid, even for days, if simply immersed and left at rest; but the action was started at once by moistening the steel with the chromic liquor and exposing it to the air in that state for a minute or two before immersion. By supporting the small platinum sieves upon funnels immersed in the liquids, the heavy solution of ferric sulphate passed down through the funnel as produced, and thus a continuous circulation of the solvent was promoted.

Preparation 1.—Four pieces of the sheet steel (from 7 to 7.5 grammes each) were exposed to the action of the solvent described in separate vessels. After the lapse of two days, there only remained

small quantities of a grey-black powder upon the sieves; this was washed off into the chromic liquor and, together with the powder which had collected at the bottom of the vessel, was allowed to remain from eight to fourteen days in the liquor, the time varying with the date at which the several experiments had been commenced. In every case there was found to be a considerable excess of chromic acid in the solution. The four deposits were afterwards transferred to one vessel; 500 c. c. of fresh chromic liquor were placed upon the combined product, which was allowed to remain in the solvent for four days at the ordinary temperature. During this time no reduction of chromic acid took place. The heavy grey-black powder, which was strongly attracted by the magnet, was then washed, first with water several times, then with alcohol, finally with ether, and was afterwards dried over oil of vitriol in a rarefied atmosphere until it ceased to lose weight. The amount of dry residue, or carbide, obtained in these operations per 100 parts of steel was 13.25. To ascertain the proportion of carbon in this carbon-iron product, the method of treatment by solution of copper chloride was first resorted to; but the substance was attacked with difficulty by the solvent, there being no action at the ordinary temperature even after the lapse of twenty-four hours.* It was therefore necessary to keep the solvent heated, to promote its action, and this may have given rise to some slight formation of carbo-hydrogen, tending to reduce the proportion of carbon found. Moreover, this carbon, when the action was completed, was obtained in so very finely divided a condition, that its collection without mechanical loss was a matter of great difficulty. For these reasons this method of analysis was abandoned, and the comparatively simple process adopted of placing and weighing the dry material to be analyzed in a small platinum boat; inclosing this in a Bohemian glass tube; burning in a slow current of dry oxygen; allowing the products to pass over heated cupric oxide; and finally absorbing and weighing, in the usual manner, the carbon dioxide and water obtained. At the close of the operation the residual iron oxide in the boat was dissolved in chlorhydric acid, and the iron estimated. By the copper-chloride process, the percentages of carbon found in preparation 1, in two experiments, were 6.83 and 6.69. The iron, estimated in the liquids (after precipitation of the copper by electricity) amounted in these two experiments to 91.29 and 92.16 per cent. By the combustion process, the following percentage results were obtained:

Carbon	7.31
Iron	90.42
Water	2.37

In order to ascertain what proportion of carbon this product would leave unconverted into carbo-hydrogen, by treatment with chlorhydric acid, from 0.5 to 1 gramme of the carbide was heated upon a water-bath with excess of the acid (sp. g. of the acid 1.10); the portion remaining undissolved was collected upon asbestos (previously ignited), washed successively with cold water, cold alcohol, and warm ether, then dried in a current of hydrogen, while gently warmed and afterwards burned in a current of oxygen. In two experiments the carbon unconverted into hydrocarbon amounted to—

1.410 per 100 of the carbide, or 20.87 per 100 of carbon in the carbide,

And

1.238 per 100 of the carbide, or 16.93 per 100 of carbon in the carbide.

Preparation 2.—Two pieces of steel, weighing about 7.5 grammes each, were treated with the particular chomic solution above described, 1250 c. c. of liquid being used in each case, and the treatment being carried on for four days. The two products, of the same nature as those constituting preparation 1, were then transferred to one vessel, and left for two more days in contact with 250 c. c. of fresh chromic liquor, which appeared unaltered at the end of that time. The amount of carbide which this treatment furnished per 100 of steel was 14.16. The analysis of the dried product by the combustion process furnished the following percentage results:

Carbon	7.21
Iron	90.64
Water	2.27

* This circumstance afforded additional proof that metallic iron had been completely removed by the chromic treatment.

The carbon remaining unconverted into hydrocarbon, by treatment of this product with chlorhydric acid, amounted to—

1.269 per 100 of the carbide, or 17.60 per 100 of carbon in the carbide.

Preparation 3.—Two pieces of steel, weighing about 7.5 grammes each, were submitted in separate vessels to the action of 2000 c. c. of the solution already described (the comparatively weak chromic liquor) for five days. The united products were afterwards left for five days in 500 c. c. of fresh chromic solution, which did not appear at all affected. The amount of carbide obtained from 100 parts of steel was 15.34. The percentage results obtained in three examinations of the product were as follows:

Carbon.....	6.84	6.84	
Iron.....	91.50	91.50	91.50
Water.....	1.63		

The carbon unconverted into hydrocarbon by treatment of the carbide with the chlorhydric acid amounted to—

0.836 per 100 of the carbide, or 12.22 per 100 of carbon in the carbide.

Preparation 4.—Here 14.7 grammes of the steel were exposed for three days to the action of 1900 c. c. of the chromic solution described (the strongest solution), and the product obtained was afterwards left in contact with 350 c. c. of fresh solution, which appeared but very slightly affected at the end of that time. The amount of carbide obtained from 100 of steel was only 4.66. It was found to have the following percentage composition:

Carbon.....	11.77
Iron.....	80.57
Water.....	5.57

There was not sufficent material for a repetition of the analysis, nor for ascertaining the proportion of residual carbon after treatment with chlorhydric acid. For comparison the amount of carbon unconverted into hydrocarbon, by treatment of the original steel with chlorhydric acid, was determined and found to be:

0.039 per 100 of steel, or 3.41 per 100 of carbon in the steel.

Table A is a tabulated view of the results obtained in these four series of experiments. An examination of the foregoing results suggests the following observations:

1. The two chromic solutions used for the production of preparations 1 and 2 (which differed but little from each other in regard to the amount of chromic acid present, and were produced with a bichromate solution saturated, or nearly so, in the cold). furnished results in all respects very similar, though the details of treatment of the steel with these solution differed somewhat. The third, a much weaker solution, furnished results which, allowance being made for the small quantities of products to be dealt with, and difficulties of their analytical examination, must be regarded as closely resembling those obtained with the other two solutions.

TABLE A.—RESULTS OF TREATMENT.

	Preparation 1.	Preparation 2.	Preparation 3.	Preparation 4.
Carbide obtained per 100 of steel.	13.25	14.16	15.34	4 66
Composition per 100 of carbide—				
Carbon...	7.31	7.21	6.84	11.77
Iron.....	90.42	90.64	91.50	80.57
Water....	2.37	2.27	1.63	5.57
Atomic ratio of iron to carbon	Fe 2.65 to C_1	Fe 26.94 to C_1	Fe 2.867 to C_1	
*Parts of carbon obtained in form of carbide per 100 of steel....	0.969	1.021	1.049	0 548
Carbon unconverted into hydrocarbon by treatment of carbide with chlorhydric acid—				
Per 100 parts of carbide	{ 1.410 / 1.238 }	1.269	0.836	
Per 100 of carbon in the carbide.	{ 20.87 / 16.93 } (Mean 18.9)	17.60	12.22	

* Had the chromic treatment given rise to no formation of hydrocarbon, the amount of carbon obtained as carbide should have been 1.144, that being the total amount of carbon in this steel.

2. The results obtained with the stronger chromic solution (preparation 4) indicate that the limit of the concentration of oxidizing power, which the separated carbide is capable of resisting, has here been exceeded. Not only has there been in this case a comparatively very considerable loss of carbon, as carbo-hydrogen (or possibly also as a soluble product of oxidation), but the iron in the separated carbide has also been to a considerable extent attacked; and but a comparatively small proportion of the carbide remains in admixture with separated carbon, the latter partly in a hydrated form, and possibly also in some partially oxidized insoluble form.

3. The proportion of combined water in the products obtained with solutions 1, 2, and 3, would seem to indicate that, in these also, the carbide exists in admixture with small proportions of a carbon-hydrate, which may be a result of the action of the chromic solutions on the carbide side first separated. This may possibly account for the not very definite, though on the whole uniform, atomic ratio of the iron to the carbon in the products of preparations 1, 2, and 3.

4. Duducting the proportions of carbon, reconverted into hydro-carbon by treatment of the products with chlorhydric acid, from the percentages of carbon in the products obtained in preparations 1, 2, and 3, the results exhibit a uniformity which, if accidental, is somewhat remarkable. Thus:

TABLE B.—FINAL PROPORTION OF CARBON.

	Preparation 1.	Preparation 2.	Preparation 3.
Per cent. of carbon in product.....	7.31	7.21	6.84
Less carbon unconverted into hydro-carbon.....	18.9% = 1.38	17 6% = 1.27	12.22% = 0.84
There remain of carbon per cent.	5.93	5.94	6.00

The atomic ratio of this residual percentage of carbon is as 1 to 3.270 of iron.

5. It will be observed from table A that the amount of carbon, eliminated in the solid form by the chromic treatment, most nearly approaches the total amount (1.144 per cent.) of carbon contained in the steel, in the case of No. 3, when the weakest chromic solution was employed—a result which was anticipated. Even in this case the two figures do not approach each other quite as closely as they did in the case of the specimen of cold-rolled steel, referred to in the preliminary report; but this may perhaps be in part ascribable to the circumstance that, in the latter case, no search was made for water, in the products obtained by the chromic treatment. It appears conclusively established that, in all instances, some portion of the carbon is expelled as hydrocarbon by the chromic treatment; and that some small and variable proportion of the carbide, separated from the cold-rolled steel by the chromic treatment, is acted upon by chlorhydric acid, with disappearance of iron and formation of some carbon-hydrate, concomitantly with the formation of some oxidized products.

6. On the whole, these results, which are in all respects more complete than those obtained in the much smaller and really preliminary experiments described in the former report, appear to furnish some foundation for the belief that the material separated from cold-rolled steel, by the action of a sufficiently dilute chromic acid solution, contains an iron carbide corresponding or approximating to the formula $Fe_3 C_1$, or to a multiple of that formula. The requirements of such a formula are intermediate between those furnished by the original percentage composition of preparations 1, 2, and 3, and by the composition of these, after deduction of the proportions of carbon unconverted into hydro-carbon by their treatment with chlorhydric acid.

The results of these experiments with cold-rolled steel of a particular composition appear at any rate to confirm the correctness of the view that the carbon in cold-rolled steel made by the cementation process exists, not as simply diffused mechanically through the mass of the steel, but in the form of an iron carbide—a definite product, capable of resisting the oxidizing effect of an agent which exerts a rapid solvent action upon the iron through which this carbide is distributed. Whether this carbide varies in composition to any great extent, in different descriptions of steel, which are in one and

the same condition of preparation (*i. e.* cold-rolled or annealed), remains to be demonstrated by further investigations, if the determination of this point is considered of sufficient importance to warrant the expenditure of the time and labor which it would involve. The preliminary experiments with small specimens of cold-rolled, annealed or hardened steel, described last year, appeared to warrant the belief that the condition of the carbide in the metal is affected to such an extent by the process of hardening, as more or less completely to counteract its power to resist the decomposing effect of such an oxidizing agent as chromic acid solution. How far this may always be the case, and how far it may be possible to prove that similar effects to a modified extent are produced by submission of steel to tempering processes in different degress, may perhaps be determined by further research in this direction.

RECENT HYDRAULIC EXPERIMENTS.

By Major ALLAN CUNNINGHAM, R.E., Fell. of King's Coll., London.

Minutes of Proceedings of the Institution of Civil Engineers.

III.

CORRESPONDENCE.

MR. FLAMANT considered the author's experiments to be extremely important, and that his book, full of practical details of the manner in which he worked, should be read by all those who proposed to undertake similar experiments, or who desired to study the phenomena, still very obscure, which occurred in the flow of water in large open channels. It was for this reason that he had referred to the work in a note inserted in the "Annales des Ponts et Chaussées," desiring thereby to induce French students of the subject to consult Major Cunningham's book. It was impossible to render too much praise to the care and exactitude exhibited by the author in his operations, and the remarks that followed referred wholly to questions of theory or of rightful interpretation.

The author said (chapter v.) that according to theory, the interior pressure of flowing water in permanent motion was less than that in the case of still water, and that it diminished with the velocity. Consequently, if a body of still water were connected by a tube of small aperture with a running stream, the level of the latter would be higher than that of the still water. He had proved this by experiment, but the difference of level shown had been very small (about 0.07 foot). Here, Mr. Flamant thought, was an error of interpretation. The tube in communication with the running stream would project from the bank. It would, in consequence, oblige the particles of water impinging against it to take a curved trajectory instead of preserving their rectilinear direction. In the path of this trajectory, of which the concavity would be turned towards the tube, was developed a centrifugal force, tending to throw the particles of water from the tube. In this way a kind of suction was set up in the tube, which would diminish the pressure, and consequently lessen the height of the water in the adjacent reservoir, making the latter less than that of the stream, which indicated the real pressure, except in the immediate neighborhood of the tube. This effect would be the more marked the greater were the centrifugal force, *i. e.*, the velocity of the stream. The author had been able to show that the difference in the two levels increased with the speed of stream, but he would not have observed any difference at all if the tube had been placed close to the slope of the stream's bank without projecting into the water.

The same would apply, Mr. Flamant thought, to the author's statement in chapter viii., that the surface of the water should be convex transversally. The author endeavored, without success, to measure this convexity. Nevertheless, as the velocity of the water at the banks was nearly *nil*, the level should be, according to his theory, sensibly that of still water,

and, logically, he should have been able to find the same difference between the level at the banks and in mid-stream as between the latter and the water in the reservoir. But between the water near the banks and in mid-stream there existed free communication, without the intervention of any tubes to interfere with the conditions of flow, so that the pressure was the same at equal depths, *i. e.*, the surface was necessarily horizontal. The instance of convexity of surface noticed by Baumgarten in the case of the Garonne did not correspond to a permanent condition, but to a period of rise in the stream, at the beginning of which water would mount more quickly at the middle than near the banks.

Lastly, in chap. xvii., the author stated that he had noticed near the banks, and on the surface, a persistent current from the bank towards the middle, which current was greatest inshore, and rapidly diminished as it approached the center. It seemed to Mr. Flamant that this current towards the middle could only be an illusion. When there was placed quite close to the bank a float, whose dimensions could not be neglected in respect of its distance from the bank, the fluid veins which impinged against it differed in force sufficiently to make the float turn on its axis, and drive it from the shore. This effect would diminish in proportion as the distance from the bank being greater the velocities of the fluid veins became more assimilated. Moreover, a current implied a displacement of liquid; it would therefore be necessary that the water flowing from the bank towards the middle should be replaced by the product of a similar current from the center towards the bank. The author thought that there really did exist such a current, for he had noticed that loaded rods placed near the bank were less acted upon than surface-floats, and that they preserved a movement sensibly parallel to the bank. This might be explained by the fact that loaded rods were endowed with a motion comparable to the mean velocity on a vertical, and that all things being equal, this mean velocity varied less than local velocities, and especially than the surface velocity. The author had himself observed that the curve of mean velocities was often more flat than the transverse curves.

These objections referred only to secondary points; they in nowise vitiated the observations themselves, which had been made with a care and good faith which must be generally acknowledged, neither did they affect the main conclusions arrived at by the author concerning the discussion of the formulas of mean velocity, and which were amply justified by the observations.

Mr. Robert Gordon remarked that the hydraulic researches published by the author of the paper at Roorkee, in 1881, formed a most important connecting link between the previous experiments of Messrs. Darcy and Bazin, and others, on small regular channels with uniform flow, and the more numerous experiments on large natural channels with varying flow, of which the elaborate investigations inaugurated on the Mississippi by Messrs. Humphreys and Abbot formed the type, and, perhaps, the most important example. Neither in his aims nor in his results did the author strive to raise any new problems; but throughout his inquiries, as narrated in his Report, he restricted himself to ascertaining what light the careful and accurate experiments conducted by himself, and involving an immense amount of conscientious labor of the highest class, could give to the problems now perplexing hydraulicians, and to placing on record for students of the science the elaborate and clearly arranged data gathered in his inquiries. The Ganges canal, at the part where the experiments were conducted, reached the magnitude of a river, both in its cross-section of nearly 200 feet wide by 12 feet deep, and in its discharge of nearly 7,500 cubic feet of water per second. It verged on the conditions of a river, also, in its varying and irregular flow; while, from the complete control over some of the principal elements influencing the flow, it retained all the advantages of an experimental channel. No person could peruse the Report without recognizing its high value as a permanent contribution to the science. It did not mark out a new era, like the works of Guglielmina or Du Buat, or those of Darcy and Bazin, and Humphreys and Abbot; but it consolidated on a firm basis the results acquired up to the present, enabled the deficiencies of the science to be seen, and might inaugurate a new departure with definite

aims and scope. A brief notice of the present condition of the science, as ascertained by the author, and generally confirmed by other writers, might be presented. Before doing this two exceptions might be taken to Major Cunningham's work, one relating to the matter, the other to the treatment. These were the only ones that occurred to him, and were suggested with diffidence. First, the Ganges canal itself did not offer, in any of the experiments taken, all the conditions for free flow. At the end of each reach of the canal a permanent weir had been constructed to check the velocity of the water; and on the permanent weir temporary weirs were placed, which had the effect of changing the character of the flow throughout the whole reach of the canal above it. These changed conditions were recognized and fully stated by the author, but the fact remained that in not a single instance was the discharge in the condition of free flow normal to ordinary rivers, and also to experimental canals. How far the analytical results were changed, or if at all, especially in the relations of the velocities on verticals, and in the comparison of the measured discharges with those given by formulas, it was impossible to say.

The second exception as to treatment referred to the use made of the method of least squares in displaying the results of the velocities on the verticals. Obviously, before it was used at all, some law must be assumed to exist and to be known, or else an arbitrary formula must be chosen, and all that the method of least squares could do was to assign the most accurate indices and coefficients to the factors already assumed in the law or formula. There was, therefore, an objection to its use before the law had been ascertained by which the order of the elements changed. The author assumed that the order of the velocities on each vertical changed like that of a parabola, and determined the particular curve in each case. He also assigned an arbitrary weight, or value, to the velocities according to their distances from the surface, when taken by the double-float, giving the lower velocities a very much inferior value to the upper ones (chap. xi., p. 4). Theoretically he was correct; but his practice appeared exaggerated, and not sufficiently supported by evidence. As the vertical velocity-curve was the most important of all the analytical results yet obtained, it was desirable to preserve it as free from alteration as possible. Probably the future science of hydraulics would take its principal direction from the teachings of this curve.

That the science needed a new direction was evident from the author's words, where he summed up the results of his comparison of the measured and calculated discharges of the canal. He tested all the best known formulas in ordinary use, and said: "The general result of trial of these formulas, which are all empirical, shows that at present increased approximation can only be obtained by increased complexity; there is no guide as to the form of such improved approximations, whilst the labor of tentative research is excessive. Until some guide is obtained from a rational theory, it seems to the author hopeless to attempt further improvement." This statement was but the echo of almost every writer of authority on the subject within the last few lusters. It had been the one leading object of almost every investigator for a long time to, in the first place, improve and facilitate the mode of research, and the means for calculation for practical engineers; but the further the inquiry was pursued, and the more accurate and elaborate the data, the wider were the discrepancies between practice and theory. Indeed, hydraulics, as it existed, was a science without a guiding theory, as the old theories had been entirely discarded by writers of judgment and experience. But there seemed a diffidence, or a reluctance, or a shrinking from the responsibility of originating a new theory, or even of examining tentative theories on the part of the investigators, to whom practical men looked for guidance in developing the resources of the science to the urgent requirements of modern life. The experienced practician by tact and judgment might overcome difficulties in the field, often at an immense expenditure of time and money, but he could not transmit his personal acquirements, nor enable schoolmen and professors to embody in their text-books and lectures the rationale of his results, unless they were prepared with a well-grounded theory to fit them in to the series of facts consti-

tuting the science. So far from throwing new light on old problems, each new investigator, from lack of a rational theory, only rendered them more obscure.

That this was no exaggerated view was shown from Major Cunningham's other results. He investigated the amount and value of the surface slope of the canal. That this slope was one of the most important elements in calculating the discharge of a stream was well known, yet this was what the author said of it:— "The general conclusion from over five hundred cases was, that surface-slope measurement is so delicate a matter that the results are of doubtful use." Mr. Gordon could not agree in this conclusion as a general one, though it seemed applicable to the Ganges canal, solely on account of the weirs at the lower end of each reach. From experiments made on the Irrawaddy, now extending over several years, he believed that it was possible, by increasing the number of points and spreading them over a considerable space in order to make continuous or daily observations, to ascertain the general slope of a river or canal with sufficient accuracy to permit it to enter into a formula as a factor. But a very few observations, neither simultaneous nor extended, such indeed as had been too often used in some published results, were most misleading, and, taken with the utter absence of theoretical guidance to indicate the degree in which they should enter into the formula of calculation, landed the user of them in a dilemma of bewilderment.

The author did not leave the discussion on the practical methods of measuring the discharge of an open channel in a satisfactory condition. For small regular channels there was not much difficulty; but for natural channels, and even for the Ganges canal, the final solution as to the best instrument for measurement had not been obtained. He favored the double-float; in this agreeing with Messrs. Humphreys and Abbot and himself. But the author did not treat the results with sufficient respect in his use of the method of least squares. He also approved of the rod, which apparently answered well in experiments like his own, and those of Mr. Francis at Lowell, in regular-shaped channels, not of great depth. But in moderate-sized channels of irregular shape, it had been found by the experimenters in the Rhine in Holland, that the rod was incorrect and unsuitable. They and other continental investigators appeared now to have entirely adopted the Woltman meter in one or other of its various forms. The author, however, seemed unequivocally to condemn these, so far as his experience extended, and quoted General Abbot with approval, who held the same view still more strongly. Mr. Gordon had for several years carried on extensive experiments on the Irrawaddy with double-floats, and had endeavored during the last flood-season (1882) to test these with three of Deacon's electric current-meters. The Indian Government gunboat "Irrawaddy," with a large staff, was at his disposal, and most elaborate tests had been made. Two of the meters had been sent to be re-tested at Torquay by Mr. Froude, who originally ascertained their coefficients at the Admiralty experimental tank, and the result was not yet known. But from a careful examination of the results of the experiments, which were conducted in depths of from 12 to 100 feet, and up to velocities of 8 feet per second, it was feared that the instruments changed their rate in the silt-laden water, which also carried an immense quantity of fibrous vegetable matter in a fine state, the lowering wire and meter often coming up covered and clogged with this.

While, therefore, he believed that hydraulicians were deeply indebted to Major Cunningham for his valuable labors, and for placing the true state of the science so conspicuously forward, it was much to be regretted that it was not found possible to bring within the scope of the work some discussion or indication of the rational theory, by which alone the present confusion could be overcome, and real progress ensured. The author had already shown in separate papers that when he took up this branch of the science he would do much to elucidate it, and when the clue was found it was hoped that the present experiments would find their true interpretation, with those of others, in building up a sound structure of history.

Mr. G. Hagen, of Berlin, held that the laws hitherto discovered on the motion of water in rivers were of no value practically. The subject was too complicated, and very difficult, and the observations had not been sufficiently exact. He had already given his views respecting the

Roorkee hydraulic experiments in "Zeitschrift für Bauwesen."

Mr. J. S. Hollings remarked that, were it possible to ascertain with accuracy the mean evaporation of the whole globe, it would most likely be found to be exactly equal to the rainfall (for rain was but condensed vapor); otherwise the sea would be either advancing or receding on all shores alike, for the ultimate goal of all water not evaporated was the sea. Living as he did in Montserrat, a small island in the tropics, he had ample opportunity of observing how very little rain fell on the sea, compared with the land. He felt sure, if correct measurements of evaporation from the sea could be obtained, they would show a large excess over the rainfall on its surface; whilst on the other hand, the rainfall in a mountainous country would be in excess of the evaporation. So many causes, however, tended to modify evaporation, that it was almost impossible to keep such a gauge correctly. The temperature in the sun in the West Indies rose to about 150° Fahrenheit on hot days, and was rarely lower than 78° in the shade, calling the shade the coolest place available for a thermometer. This latter was also the mean temperature of the sea, whilst 80° was that of the air over the land in the coolest place. On some days with little wind, the wet bulb of the hygrometer in the shade was only 1° less than the dry, whilst on other days, with the same sun-heat, 10°, and occasionally 12°, difference were registered, the only altered condition being the movement of the air. The evaporation from equal surfaces of water on sea and land would most likely also differ materially under the same conditions of sun-heat and air movement, for the vast reserve of unheated water in the sea would continually retard evaporation from the surface water, whilst the comparatively thin stratum exposed to the sun in lakes, rivers, canals, reservoirs or pools, would soon get heated through and rapidly evaporate. It was therefore almost impossible to obtain sufficiently reliable results from any number of evaporation gauges on land, or floating in terraqueous suspension, to form a basis for calculating the mean evaporation of the globe. He believed an evaporation-gauge had been kept at Barbadoes for several years, and that the average evaporation was about 3 inches per month, which agreed with the author's $\frac{1}{10}$ inch per day. The maximum evaporation was, he thought, under 4 inches in any month; whilst the average rainfall was about 54 inches per annum. The hygrometer readings in Montserrat agreed pretty closely with those at Barbadoes, whilst the rainfall at sea-level was about 46 inches; at 250 feet above sea-level, 56 inches, and at 1,200 feet above, 84 inches. Thus, wooded mountains of only a little over 1,000 feet elevation, led to nearly double the rainfall, whilst at the same time they probably lessened evaporation by more than half; for at the higher elevation he had never seen a difference of more than 5° Fahrenheit between the wet and dry bulb, and rarely more than 2°. All this tended to prove that, until a series of observations embracing different grades of level of land, and varying depths of sea, both for evaporation and rainfall, were collected, the question of whether the one was in excess of the other could not be answered.

Mr. James Leslie observed that the experiments showed a smaller velocity at the surface of the water than at some depth. Might not this arise from the floats being slightly above the surface of the water, and so being impeded by the air, if calm, or still more if there was some wind in a direction opposite to that of the water? The average velocity seemed to have been taken with great care from many points at different depths, and at different horizontal distances, and if the sectional areas were given, would afford accurate data for calculating the discharges. The surface fall in any ascertained length might have been given with advantage, and either two or more cross-sections in that length, or the sectional areas and rubbing surfaces, so as to afford means of ascertaining the mean hydraulic depth, and thereby the proper coefficient for any formula adopted, for arriving at an approximation to the velocity and discharge of any river, without having to go through the troublesome operation of finding the average velocity. He thought the alleged difficulty in ascertaining the surface-level, owing to the undulations, might have been obviated by having a vertical tube or cylinder, with a small orifice at the bottom like a marine barometer or a tide gauge.

The experiments on the evaporation

from the surface of water, showing it to be only $\frac{1}{10}$ inch per day, were important, as correcting a widely entertained notion that in Great Britain the evaporation from the surface of water was much greater than that stated, and even considerably more than the loss from agricultural or pasture land.

OBSERVATIONS MADE AT FERNIELAW, COLINTON ON THE NORTH SLOPE OF THE PENTLANDS AND 500 FEET ABOVE SEA-LEVEL.

Date.	Rain Gauge.	Loss from Surface of Water.	Loss by Deep Drainage.	Loss by Surface Drainage.
	Inches.	Inches.	nches.	Inches.
1871	35.2	19.4	16.4	20.4
1872	49.1	13.0	19.4	18.2
1873	34.4	12.8	15.5	14.7
1874	32.0	18.0	18.7	18.9
1875	34.1	14.2	17.6	17.0
1876	42.9	15.4	19.7	19.3
1877	45.7	15.2	17.1	16.3
1878	34.4	18.9	20.0	22.2
1879	39.1	12.7	15.9	15.5
1880	34.4	14.5	18.7	18.3
Average	38.13	15.41	17.90	18.08

RAINFALL, AND EVAPORATION FROM THE SURFACE OF WATER IN A TUB 6 FEET IN DIAMETER, AND THREE FEET DEEP AT GLENCORSE FILTERS ON THE SOUTH PENTLANDS, 650 FEET ABOVE SEA-LEVEL. THE LEVEL OF THE SURFACE OF THE WATER WHEN SET AT ZERO IS 1 FOOT BELOW THE TOP, THUS MAKING THE DEPTH ONLY 2 FEET.

Date.	Rain.	Evaporation.	Date.	R in.	Evaporation.
	Inches.	Inches.		Inches.	Inches.
1857	34.90	12.30	1869	34.15	14.15
1858	28.75	10.90	1870	27.20	11.90
1859	35.05	12.60	1871	34.45	10.30
1860	30 00	9.50	1872	52.20	9.25
1861	38.50	10.00	1873	36.30	10.15
1862	42.80	10.25	1874	37.75	11.60
1863	38.90	12.45	1875	37.90	11.95
1864	35.75	12.15	1876	45.35	12.11
1865	35.60	9.50	1877	54.30	11.05
1866	37.50	9.10	1878	38.40	13.95
1867	37.75	10.45	1879	46.75	11.65
1868	46.00	14.70	1880	45.00	12.10
			Average	38.80	11.42

He had found, from many observations, that in dry summer weather, evaporation from a surface of water in Scotland was only $\frac{1}{12}$ inch per day, and throughout the year it was considerably less than the loss from the surface of land having vegetation on it, showing that it was a mistake to deduct the surface of a lake or reservoir from that of a gathering ground.

As the Indian observations on the evaporation from a surface of water were generally made during dry weather, those observations during wet weather being mostly rejected, it might be assumed that the average evaporation for the whole year would be much less than $\frac{1}{10}$ inch per day.

The evaporation was from the surface of water in a vessel about 1 foot in diameter, and from two zinc tubs about 2 feet in diameter, filled with earth and growing grass, the one outlet being at the surface representing undrained land, and the other at the bottom representing drained land.

MR. R. E. MCMATH, of St. Louis, Mo., U. S. A., observed that the theory upon which slope-formulas were based required uniformity of cross-section as an indispensable condition. The longitudinal section, Plate 1, "Roorkee Hydraulic Experiments," showed that the immediate vicinity of the experimental sites did not satisfy this condition. Therefore verification of slope-formulas could hardly be expected from the experiments, and it would not be fair, upon their evidence, to press the conclusion that empirical formulas were of "uncertain applicability." Limited in application they certainly were.

Likewise the underlying theory of discharge tables did not allow of change in the conditions of out-flow from the reach. The experimental sites were subject to the influence of the "State of control," and consequently observations should alone be compared, which had been made under similar conditions of obstruction in distributaries and at the tail of the reach. Any change in these conditions required a new table rather than complication by "double entry."

As a first impression, it might seem that the tendency of the work was to discredit formulas and discharge-tables. A truer interpretation of its teaching was that both must be used within rigid limits, and not loosely as had been the practice. Both methods were useful when properly applied, and were indispensable to the engineer, the one when designing, the other for continuous-discharge deter-

mination. But this necessity attached to essentials, and not to forms.

The author said in regard to formulas, "Until some guide is obtained from a rational theory, it seems hopeless to attempt further improvement." An important step towards a rational theory and practice would be effected when recognition was taken of the wide difference between an artificial channel of unvarying section and uniform bed-slope, and the natural stream in which both varied continually. The use of a formula containing slope and hydraulic mean depth was strictly limited to the former. Otherwise $S=\frac{\text{Fall}}{\text{Distance}}$, and $R=\frac{\text{Sectional area,}}{\text{Wet border}}$ were not properly associated together, S being for a distance, and R for a single point in that distance. To be logical

$$\frac{\text{Cubical contents of reach}}{\text{Wetted surface}}=\rho$$

should be taken instead of R. In such case the particular section chosen for a site should be that in which $R=\rho$, the latter being determined for the full slope-length. This involved another condition. The local values of R must either increase or diminish throughout the slope-length, in order that ρ might be a hydraulic mean, as distinguished from mere arithmetical mean.

These conditions had not been observed even by the originators of slope-formulas, and yet, he thought, need only be stated to be accepted as controlling.

In a natural stream the unequal distribution of fall suggested that surface gradient, local slope, measured the retarding rather than the accelerating force, answering to the force expended in overcoming resistances, rather than to that which produced motion. Momentum and facility for transmission of pressure had to do with motion in open as well as in inclosed channels, for the current often continued when slope was negative. A rational theory recognizing these facts would have regard to head and not to its accidental distribution.

The possibility of useful discharge-tables—and there were such—was so closely connected with the better theory, that to show the conditions of the regular law of discharge implied in a table was, so far as it went, the development of the theory.

1. For a discharge-table, it was essential that the preliminary observations should be made at the site for which the table was to be prepared; or that simultaneous gauge-observations should be made and referred to a common datum, so as to refer each observed discharge to the proper local height.

2. The cross-section should be a regular figure, at least above the level of no-flow, preferably a rectangle.

3. There should be a considerable wet area remaining when the water was drawn down to the lowest possible level. For area controlled the increments of velocity, and determined a discharge-curve, which, if not simpler, was at least flatter, and therefore determinable with less percentage of error than when area became small. This condition located the discharge section in a "marked hollow in the bed-slope," and required it to be in the obstructed sub-reach.

4. In the obstructed sub-reach, the crest of the fall defined the level of no-flow. In a river, the crest of the shoal defined that level in the reach above. This level was different from the plane of low water. It fixed definitely the origin of the discharge-curve.

5. The conditions of discharge from the reach must be invariable. The utility of discharge-curves rested upon their affinity to a weir-formula; or that the discharge over a given weir, whether free or submerged, was for a given head a definite quantity. If the opening at the weir were changed, in any way, the discharge would be affected. The opening of side-channels (distributaries) might be the equivalent of lowering the crest, as well as of increasing the area of discharge.

6. The conditions of approach to the reach should be such, that currents should follow the same general direction at high and low stages, and that no eddy should at any time exist at the discharge-section.

7. In a river it was important to select a site remote from a tributary, if possible, but below rather than above.

By observing the above conditions the results would be reliable if the local law of discharge was well determined.

For such determination a series of observations would be necessary, covering a range of stage as wide as practicable. The conditions stated were intended to make the discharge conform to an equa-

tion of the second degree. Since adopting the author's notation, $D=AV$, an equation of the second degree implied that the varying values of A and V should each follow the law of a straight line. In a rectangular section this was secured for A.

$$A=bh.$$

Also if the section was irregular below the level of no-flow and rectangular above, for the area below no-flow level might be represented by a constant a, and

$$A=a+Eb.$$

E was the elevation of surface above the level of no-flow.*

For the velocity-law recourse must be had to the discharge at the fall, and to the conditions of motion in the obstructed sub-reach.

The head of the sub-reach was defined by the intersection of the line of no-flow with the "thalweg" profile. Taking the elevation of the surface at the head of the sub-reach above the crest of the fall (the weir, not water-crest), E was obtained, the head which controlled motion in the reach. A certain part of E was consumed in overcoming resistances; the remainder, $E_1-E_2=E_s$, was found at the fall where the discharge D, definite for a given value of E_s, passed through a section of fixed dimensions A, with a mean velocity V, due to the fall E_s. At no other point in the sub-reach was there a velocity approximating that due to the fall, local or aggregate, being clearly too great for the local $\triangle E$, and too small for the aggregate E. At any section with an area A, the mean velocity V, would be the quotient of $\frac{D}{A}=V$, and $\frac{V_1}{V}=\frac{A}{A_1}$. This relation of velocities at the section in the reach and at the fall held at all stages; consequently if there was a law at the fall, there must be at every section in the reach a direct relation between E and V.

If consideration was given to the increments of V at any section for definite increments, e, of E, evidently $\frac{A}{A+eb}$ and $\frac{A+eb}{A+2eb}$ would differ but slightly, and by a nearly constant difference, if the initial value of A was large. This was provided by the third condition, therefore the actual values of V would be closely approximated by the equation of a straight line,

$$V=cE+d.$$

Combining with $A=a+bE$, then

$$D=Av=cbE^2+E(ac+bd)+ad.$$

For the coefficient c, and constant d, the measured discharges must be depended upon.

Within the ordinary range of measured discharges it would be found that, the better the observations, the more closely would the mean velocities approach a straight line. But if accuracy was reached in the first decimal place, the fact that the law of mean velocity was of the second degree became apparent. The curve was very near a hyperbola,

$$V=\left(\frac{A^2}{B^2}E^2+A^2\right)^{\frac{1}{2}}\pm A,$$

A being the transverse axis, and an apparent function of $\sqrt{2gE}$, B being the conjugate axis and an apparent function of mean depth, or hydraulic mean depth.

Since curves were by observation concave or convex to the axis of E, according as mean depth was less or greater than an undetermined limit, it was suggested, that when the hydraulic mean depth of discharge section was equal to

$$\rho=\frac{\text{Cubic contents of reach}}{\text{Wetted surface}},$$

the velocity law was a straight line.* This furnished another condition of choice for a permanent discharge site.

8. The local value of R should approximate that of ρ for the obstructed sub-reach.

According to the view taken, the section at the crest of the fall was the limiting section for the sub-reach; but this was true only as it was the section of least area. For the head, E, in the reach was that required to force the discharge through the smallest area. The relations shown would be destroyed, if the site cho-

*In a paper read before the American Society of Civil Engineers, No. ccxxxix., vol. xl., Transactions, 1882, this element, new to hydraulics, was named "ruling-depth," and was represented by $\triangle$, but as that symbol was appropriated, Mr. McMath therefore now substituted E, an abbreviation for "elevation above weir-crest."

* If this suggestion was not strictly correct, it was at least an approximation and of value as a practical guide in selecting a site.

sen was above a part of the reach so narrow that the area at high water might be less than at the crest of the fall. The limiting section must be least in area and depth. The argument had followed the case of a dam of free overflow, in order that the definite value and influence of E might be unmistakable. The relations would be no less useful if the dam was submerged, or if it was a natural shoal.

Hints of the more rational theory had appeared in the foregoing discussion. It might therefore be appropriate to add, in closing, that the better theory could only be reached and tested by hydraulic experiments, whose results should be cleared of the effect of "unsteady motion." This could not be done by any method of direct velocity measurement; but could be by making an absolute measure of the quantity discharged in a given time. Mr. J. B. Francis, in his Lowell experiments, obtained mean velocities accurate to the fourth decimal place, as was evidenced by systematic residuals when compared with approximate computed curves, good to the second decimal place. With such data to work from, the way to a rational hydraulic theory would not be long.

Mr. John Neville thought the measurements to obtain the average velocities and depths (pp. 7 and 8) should have been extended for longitudinal as well as for cross-sections. The width should have been divided into several sub-runs or sub-channels, moving side by side according to circumstances, but not less than three. Such measurements would give a simple and sure operation for finding the discharges from top to bottom within the widths of each sub-run; and between the banks, by adding the sub-discharges together (similar to the use of offsets to a chain line in surveying), which would obviate the use of formulas (p. 27), by suitable admeasurements.

The means of determining the bed, side, and mean velocities was not very clear. The author said (p. 25), "The decrease of forward velocity is so rapid close to the banks as to make it clear that the forward velocity must be very small:" "it does not admit of direct measurement." And again, "In a float-course only $7\frac{1}{2}$ inches from a straight vertical bank one hundred surface-floats were run before three (3) were obtained in fair course over a $12\frac{1}{2}$-feet run." Mr. Neville had himself often found a backward surface-flow near the banks of rivers. It was stated (p. 10) "that the range of velocities, deduced from a number of similar floats run in rapid succession over nearly the same float course, was commonly 20 per cent. of the mean. In some of Harlacher's experiments a current-meter was fitted with electrical connections so as to record every revolution; the variations amounted to from 20 per cent. in surface-velocities to 50 per cent. in bed-velocities in a few seconds." He had often observed that the flowing section of a river did not always correspond with the section of the channel taken to the water-level. Yet an average forward discharge took place, notwithstanding many changes of direction in the motions. Practically, taking the mean velocity as v, and the maximum surface-velocity as V in feet, the formula deduced from Du Buat's experiments by Prony, viz.,

$$v = \frac{7.783 + V}{10.345 + V} \cdot V,$$

gave more consistent practical results for all channels in the run of professional work than the more complex formulas of later writers on the subject.

The falls, of about 8 feet, at the lower ends of the reaches on which the observations were made, must have affected the results, and depressed the surface maximum velocities down towards the level of the crest of the overfall or sill of sluices. Great disturbance must also have existed below the foot of an upper, and head of a lower, reach. The depression of the maximum surface-velocity from wind and atmospheric causes was only occasional and partial, although the earth and the atmosphere formed a sort of compound tube for the water section to flow in. Notwithstanding that the vertical velocity-curves were "all very flat" (p. 16), so flat that "any geometric curve could be fitted very close to them" (p. 17). The statement, therefore, that the "last result is of great scientific importance," viz., that "the vertical curve is nearly a common parabola, the error of whose computed parameter is often very large," was scarcely justified. Also the statement that "no confidence can be placed in values of the parameter not formed by the method of least squares;" concluding with: "After many attempts

to construct a new formula, the conclusion is drawn that the data are too uncertain to admit of it!" Page 18 would seem to show that these experiments were productive of little useful result.

That the transverse surface-curves (pp. 14 and 15) must have their convexity or concavity always small, dependent on the depths below and corresponding surface-velocities, has been well known for a long time. The great number of Roorkee experiments added nothing new to the scientific knowledge of these curves.

Observed levels of the water-service at and above the falls, in the stretches, would have given useful data for the construction of the backwater curves of much more practical and scientific value than any vertical-velocity or transverse surface-curves.

The surface-slope or longitudinal inclination of rivers was apt to lead to a great deal of misapplication of hydraulic formulas. It was so flat that, unless at rapids, a considerable change was not always apparent. The range in these observations is from $1\frac{1}{2}$ inch to 30 inches per mile, or from 0.0284 to 0.568 inch, nearly, in a float-run of 100 feet. The levels themselves varied 0.07 foot, or 0.84 inch in the transverse section (p. 8). "From twelve trials of slope-lengths of 2,000 and 4,000 feet, symmetrically situate about the same site," the slopes differed 25 per cent. (p. 12), and slopes at opposite banks differed 50 per cent. (p. 13). To find the value of the slope and for what run to take it so that it should correspond with the hydraulic inclination was practically next to impossible. Mr. Neville often found wind to affect the bank water-levels; and on the callow lands along the Shannon between Athlone and Portumna, he had observed sometimes a difference up to $1\frac{1}{2}$ inch at different times on each bank. There is an undulating wave-surface and ripple on rivers of any depth which affects the forward motion, causing it to vary from time to time in the same transverse section, and even simultaneously in different ones, the slopes of varying very considerably, although the difference in level may be small. Slopes $1\frac{1}{2}$ inch and 6 inches in a mile gave only about 0.3 inch and 1.2 inch in over 1,000 feet, yet the small difference, 0.9 inch, in this distance involved the doubling of the velocity. He remembered a lawsuit in which one engineer, calculating the flow in a mill head-race from the surface-slope, and another from floats, differed as 3 to 1. Not only did the surface-slopes vary in the same river, but so also to the depths, including the hydraulic quantities r and s and the area of the section in motion.

Mr. Neville had considered elsewhere* the various formulas enunciated by their authors from Du Buat to Weisbach and since; most of which were applicable to pipes and rivers, properly understood, r and s being factors in all. In pipes there was no limit to the inclination from flat to vertical; for canals and rivers it never exceeded a few feet per mile. The experiments from which those formulas were deduced were select and varied, ranging from pipe-diameter $\frac{1}{2}$ an inch and less to 18 inches, to open channels, large canals and rivers. There were differences, and would continue to be, between a formula grasping so great a range, in size and inclination and particular experiments; but this was of no practical importance. Very little had been done since to improve this practical branch of hydrodynamics. A formula should be framed meeting any particular group of hydraulic experiments working within practical limits; it must be in part empirical, not too complex, and capable of application to vertical short pipes as well as flat long rivers. Du Buat conceived, and first constructed, a formula to meet these conditions, using one hundred and twenty-five experiments, carefully corrected for the entrance-velocity on both closed and open channels, pipes, and rivers. Nothing better had been done since.

The results to science of his hydraulic experiments was stated by the author to be "that at present increased approximation can only be obtained by increased complexity; there is no guide as to the form of such improved approximations, whilst the labor of tentative research is excessive. Until some guide is obtained from a rational theory, it seems to the author hopeless to attempt further improvement. "A guide," and "rational theory" were given by Du Buat long ago. The surface-slopes of open channels of any good size were so small in fall, and

* "Hydraulic Coefficients, Tables, and Formulæ." Second Edition, pp. 182 to 226. Also Third Edition, pp. 188 to 251.

varied at the same time so much in ratio, that they would never be used by competent engineers or by experimenters to determine or test the observed mean velocity by the form of equation

$$v = m\sqrt{rs},$$

or any other way; though they might from the equation

$$s = \frac{v^2}{rm^2}$$

determine the slope where experiments would fail. Besides, the surface-slope in most cases was not the hydraulic inclination, nor was the average slope in 2,000 or 4,000 feet, more or less, that also found at intermediate points or at the float runs from 12½ to 100 feet long. It rested with the experimenter to determine accurately the values of s, r and m before giving an opinion on the value of any formulas containing these elements.

Mr. T. W. Stone considered that the results given in the paper did not admit of dispute, as they agreed generally with received opinion on such questions. There were, however, a few points on which he desired to remark, solely with the desire of eliciting further information. The primary objects of the experiments were, he apprehended—

1. The discovery of a good method of discharge-measurement.

2. Testing the applicability of known mean velocity formulas.

3. The discovery of a good approximation to mean velocity.

The two first objects had been attained. In regard to the third, he thought that although a practical result had been achieved in regard to existing canals, the like could not be said in reference to projected canals, or the investigation of floods in rivers, beyond the statement that Kutter's formula for variable co-efficients, applied to the old expression $C\sqrt{RS}$, would give results seldom exceeding 7½ per cent., and that such results fall far short of that given by direct discharge-measurement. In the concluding paragraph of section XXI., "Discharge-Verification," a 5 per cent. difference was called a close approximation, and it was shown in Section XX., "Mean Velocity," that Kutter's formula would give results seldom exceeding 7½ per cent.; it followed, therefore, that the author's experiments tended to establish Kutter's formula as a close approximation for mean velocity in canals and channels 200 feet wide and under. Now, Kutter's formula in effect proposed the use of the old formula $v = c\sqrt{R\,S}$ with variable coefficients, and to obtain such coefficients the two variable factors S and R were used. The formula agreed nearly as well as the discharge-verifications, yet the author said, Section XIV., "It was found that the direct measurement of any velocity was far more likely to give an approximation to this mean velocity than any expression yet known not involving velocities." And again, "It seems, then, that at present the direct measurement of velocity, such as the central mean or central surface, is more likely to give a near value of the mean velocity than any formula involving surface-slope." It should be constantly borne in mind, that with factors R and S Kutter's formula for variable coefficients agreed to 7½ per cent., while tests for discharge-verification varied, on the author's showing, 5 per cent. and more. A wide range of experiments would be necessary to determine what the central surface or central mean velocity was likely to be in any proposed channel without use of formulas; and such experiments must extend to channels of not only different sizes, but of all various inclinations and descriptions. It was necessary, therefore, that some formula should be used for the design of projected channels, the investigation of floods in rivers, &c., &c., and Mr. Stone maintained that a set of variable coefficients of mean velocity of discharge should be used in accordance with the circumstances of each special case, and the nearest similar recorded observation that could be obtained. This should be so with all hydraulic calculations. There was no necessity to alter the old theoretical formulas; the coefficient expressions might vary as experience became enlarged. Such expressions as

$$v = C\sqrt{R\,S},\ v = C\sqrt{R(f^2)},$$

used with a variable coefficient determined on the lines proposed by Kutter, would give fairly approximate results. The importance of using variable coefficients was pointed out in Mr. Stone's recent work, "Hydraulic Formulæ," and was he thought, always followed by

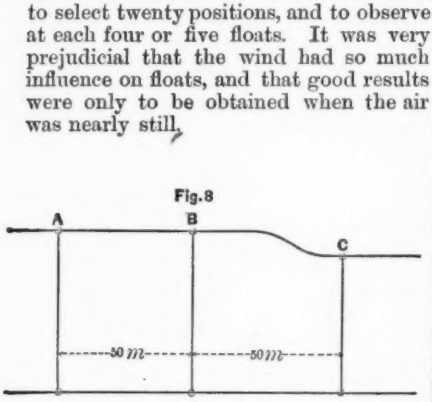
engineers experienced in hydraulics. The author's experiments had been valuable in showing that such a course led to fairly approximate results. The channels constructed by Mr. Stone in Australia were calculated from the formula $C\sqrt{R(2f)}=v$, and C was varied from 35 to 75, to give mean velocity in feet per minute, according to the nature of the proposed channel. The channels varied in section from 2 feet square to 6 feet at bottom 6 feet deep, and slopes of 1 to 1 and 1½ to 1, and some of the channels had a circular lining; the inclinations varied from 2 feet to 10 feet per mile, and the formulas gave very fair results. To establish the statement of the author, above quoted, further confirmation would seem desirable. He thought that the assertion in Section III., "As the canal water is often full of silt, this shows that the water is in pretty rapid motion close to the actual bed, and disproves the idea sometimes advanced that an obstruction across a channel causes a still-water pool above it roughly flush with its crest," should be accepted with caution. Whilst Government Engineer in Australia, Mr. Stone built a series of obstructions from 8 to 11 feet high across a channel for the same purpose as the falls on the Ganges Canal were built up, viz., to reduce the velocity. The channel in question was about 30 feet wide at the top, and irregular in section, on account of the previous velocity having been too great for the bed and banks. The large amount of silt which accumulated in the reaches above the obstructions in this case went to show that the author's deduction was not invariably correct; possibly the frequent exercise of the control spoken of at the falls for working the canal might have tended to produce scour in the bed, and so prevent silting. At Section VI. the author first referred to the fact that both parallel motion and steady motion did not exist even approximately in flowing water, and assumed in effect that all formulas based on such motions must be incorrect. Finally, however, he admitted that there was an average steady motion. Though of opinion that formulæ constructed on the theory of steady flow would be fair approximations in connection with variable coefficients, he agreed with the author that any single velocity-measurement would be clearly an accidental value. At Section VIII, "Surface-convexity," the author stated that the fair conclusion seemed to be that "the water surface across is probably level on the average." As a general rule, Mr. Stone thought the cross-section of a stream showed a horizontal line for the surface of the water only when a uniform depth and velocity obtained right across, or nearly so. If one part of a channel were much deeper than the rest, and the velocity of the water was much greater in that part than in the other parts, the level of the water became raised there, so that the surface line became uneven. When the stream was confined to only a portion of the channel, the rest being still, or backwater, the irregularity of the surface was greatest. This was exemplified by observations taken during an investigation of the floods in the river Barwon, in Australia, in 1880. It had also been observed in some of the large drains in Melbourne, Australia. According to Mr. Chulcheth, the velocity of the stream of the Barwon was 5.4 feet per second, and the river 1,400 feet wide. The mid-stream level was convex with regard to one bank, and concave with regard to the other. The Melbourne drains spoken of were about 9 inches wide at the bottom, with sloping sides of 1 in 5 on one side, and 1 in 10 on the other. They sometimes ran from 1 foot to 1 foot 6 inches deep, and had varying inclinations.

Professor von Wagner did not possess the complete work of Major Cunningham, but only the paper read before the institution, which treated in separate chapters the principal conclusions, and in some cases dealt with them more fully. It would, therefore, be simplest to give his opinion in the order of those chapters, and to add in conclusion some remarks related to the subject.

IV. He agreed with this entirely, as regarded surface floats. Large bodies, such as boats, floated, it was said, with greater velocity than the water, but small bodies, such as spherical or disk floats, might well be used for the measurement of normal velocity, because, as was proved by all experiments, no sensible acceleration could be detected. Of many experiments on this point he would only name those on the Rhine at Speyer, in which two carefully adjusted instruments, a screw current-meter and a Darcy tube,

both gave a velocity of $v=2.17$ meters, while nine surface-floats, with a path of 100 meters, gave $v=2.18$ meters. To get accurately the horizontal curve of surface velocities he selected in the transverse water-line not too many positions (for a float-path of 100 meters, about seven or eight), and then for each position he observed at least ten to twenty floats. He considered this better than to select twenty positions, and to observe at each four or five floats. It was very prejudicial that the wind had so much influence on floats, and that good results were only to be obtained when the air was nearly still.

Fig. 8



Further, he thought it important to consider the longitudinal profile of the water surface (Fig. 8). If, for example, the surface velocities at a cross-section B were to be ascertained by a number of floats, the path of the floats being about 100 meters (A B = B C = 50 meters), then it might happen that in one-half of the path B the fall might be greater than in A B. If the surface had such a form, then, in his opinion, the float-path should be shortened and the number of floats observed should be increased, in order to obtain a mean velocity.

As to those floats which reached deeply below the surface (loaded rods or coupled balls), he had hitherto not been convinced that they gave accurate results. His objections to them were stated in his "Hydraulic Experiments on the Weser, Elbe, and Rhine," p. 3. Whenever accurate measurements were required he preferred good current-meters, such as those of Amsler-Laffon, Harlacher, and others. But of floats, he only used surface floats, and these only to check the coefficients of the current meters, or to obtain approximate values as indicated below.

V. Agreeing with this, he would only add in regard to the measurement of the slope, that, if there was a rippling surface, he had always made a small side basin close to the river, in which the surface was kept still by a floating plank, or even by pouring on some oil, and thus the reading was made more definite. He was led to do this by the opinion that the water-level so obtained would be a correct mean between the wave-crest and wave-trough.

He quite agreed with VI., especially with the final words "taking" to "motion." As to theory in the proper sense of the word (hitherto everything was but empirical formula), it could not concern itself with eddies, &c., but must be based on normal conditions. The chief difficulty was the necessary modification by experimental values, coefficients, &c.

His views also coincided with those expressed in VII. Whenever the longitudinal profile of the water-surface had the form of a long curve, then, according to Harlacher's method, the true slope was the tangent to the water-surface at the intersection of the wave form with the cross-section considered.

He had found as a rule that the longitudinal slopes were the same at both banks, if the river at the site had a normal bed and a straight direction. When there were irregularities, then two levelings, one at each bank, were sufficient; but with large rivers one might be desirable in the middle, *i. e.*, in the thread of the stream; its measurement was, however, exceedingly difficult.

VIII. He had also had an opportunity of observing the transverse water-line, and had given the particulars in "Hyd. Untersuch.," p. 42.

X. The statements confirmed what he had found in other rivers:

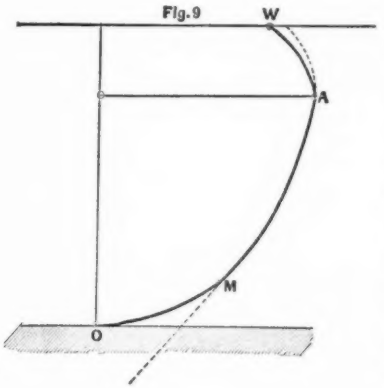
(*a.*) That generally the vertical curves were more bent, the smaller the depth of water, and *vice versa*.

(*b.*) That the mean velocity at a vertical was smaller than that at the half depth.

XI. Differences of opinion still existed as to the position of the parabolic axis. Some placed it vertically, with the vertex of the parabola at the river bed. The majority placed the parabolic axis horizontal, and at the point of greatest velocity. It was interesting to him that

Major Cunningham placed it horizontally, as he did himself. His experiments, giving a considerable number of carefully measured vertical curves, proved that with a vertical parabolic axis were obtained results altogether useless, because the normal parabola deviated so quickly from the measured curve, that there was no dependence on the position of the vertex at the river-bed. On the contrary the results were satisfactory for the other position of the axis. In his investigation of sixty-four curves of small streams and large rivers, he had come to the following conclusions:

(*a.*) Down to a point at depth M, Fig. 9 (on the average at 0.8 of the whole depth), the curve consisted of a parabola



A M, but below M, of a curve deviating from the parabola, the water being retarded by the bed. A similar deviation, though to a smaller extent, was found from A to W, possibly in consequence of the friction of the air. The vertical curve thus consisted of three parts, of which the middle one, which was parabolic, had the greatest length.

(*b.*) The axes of the parabolas were horizontal (or strictly parallel to the water surface).

(*c.*) The maximum velocity (or parabolic axis) was at from 0.0 to 0.28 of the whole depth below the surface. In the Mississippi, at 0.3. Even those curves, which were measured in a perfect calm, showed the parabolic axis below the water surface. More details of this would be found in "Hyd. Untersuch.," pp. 39, 40, and plate 8, Figs. 86 to 86 c.

XIV. The formulas here given he had compared with the values in a number of different measured curves (Weser, Elbe, Rhine Danube, &c.), and he had arrived at the conclusion that the second formula,

$$U = \tfrac{1}{2}(v_{0.211\,H} + v_{0.789\,H}),$$

gave more accurate results than the other. In most cases it agreed perfectly, the differences being never more than $2\frac{1}{2}$ per cent.

With regard to the other statements in Section XIV., as to the position of the mean velocity on a vertical, he would refer to p. 37 of his "Hyd. Untersuch." It was there shown how little the position of mean velocity differed in the most different curves. Now, if to these results were added those on the Mississippi, as well as that given in Section XIV. of 0.62 or $\frac{5}{8}$, the following figures resulted:

River. With Breadth in Metres.		Observer.	Number of Measured Curves.	Depth of mean Velocity Arithmetic Mean of the slightly varying values.
	Meters.			
Weser	80	von Wagener	6	0.587
Elbe	114	"	7	0.594
Rhine	215	"	4	0.589
"	215	"	1	0.600
Oker	14	"	3	0.600
Elbe	120	Harlacher...	9	0.620
"	120	"	10	0.598
"	120	"	9	0.580
Danube	425	"	15	0.599
Mississippi		(Humphreys and Abbot)	..	0.580
Ganges Canal		Cunningham	46	0.620

The arithmetic mean of these eleven values was 0.597.

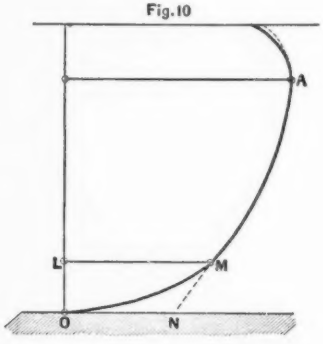
If the question was simply to ascertain the discharge for an engineering purpose then he thought it would be accurate enough to take five to seven verticals on the cross-section of a river and to ascertain the velocity on each vertical at 0.6 of the depth, measured from the water-surface.

To this he would add that the maixmum velocity in the sixty-four curves, given in the Table, varied in position from the surface (for example in the Weiser) down to 0.25 of the depth from the surface (as in the Rhine), and yet the proportionate

depth of the mean velocity differed but little.

XVII. Perfectly agreeing with these explanations, he begged leave to add the following remark. He was of the opinion that at the bottom the velocity must be zero. From that point the velocity increased rapidly. The idea of "bottom velocity" was therefore very indefinite. In practice it was frequently stated, and its value was taken as equal to O N (Fig. 10), obtained by prolonging the normal parabola A M to meet the bed, although it was proved that the velocities in the part L O decreased quicker than was given by the parabolic law. In his opinion "bottom velocity" meant only

Fig. 10

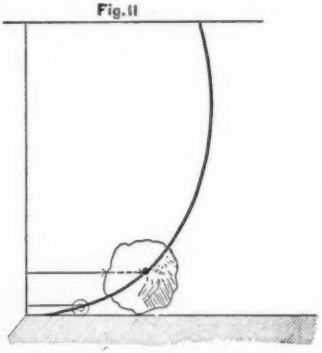


that velocity which existed at the height of the center of gravity of the boulders, on which the local average size of the boulders depended. If at any place the boulders had an average size of 10 centimeters, then there the bottom-velocity was greater than at a place where the bottom was formed of sand of an average diameter of 2 millimeters (Fig. 11). It was therefore fruitless to seek a relation between the bottom and other velocities. As to the measurement of bottom-velocity, the current-meter was not very suitable because of the distance which must be allowed for the rotation of the screw-blades. By Darcy's gauge measurements could be made at $1\frac{1}{2}$ centimeter above the bottom, provided it was level. This was shown by the curve Fig. 86 on Plate 8 in the "Hyd. Untersuch.," where, at each of the given depths, the difference of the water columns had been observed thirty times.

XVIII. Here the remarks confirmed the opinions in "Hyd. Untersuch.," pp. 40, 41, on the approximate conformity of the horizontal velocity-curve with the form of the wetted border, which was illustrated in Plate 8, Figs. 87 to 93. It would be seen that the conformity (which he had in vain sought to express algebraically) ceased in the same river with increase of discharge (Figs. 92, 93), and besides differed near the shores (Figs. 87, 89, 93), in analogy with the alteration of the vertical curves.

XIX. He considered Harlacher's the simplest and most accurate method of determining the volume of discharge. It was explained in "The Measurements of

Fig. 11



the Elbe, Danube, &c.," by H. R. Harlacher, 1881. He had used the same method in his "Hyd. Untersuchungen," p. 18.

XX. The valuable work of the author confirmed what Professor von Wagner had said, on p. 33 of the "Hyd. Untersuch.," about the degree of reliability of different formulas, and the statements in the Table, p. 32.

As the measurements in the Ganges Canal had been conducted with great care, it seemed desirable to make use of them to test two laws given in the "Hyd. Untersuch." If a continued confirmation of these laws resulted, then it would be of importance to practical engineering.

Let,

v=mean velocity of the whole cross-section.

c=maximum surface-velocity in the whole transverse width.

v_s = velocity at the center of a figure of a cross-section.

(I.) *Relation between v and c.*—In twenty-four different gaugings of small and large streams, he had found

$$v = a\,c + b\,c^2.$$

The contestants a and b were only discussed in a preliminary way in "Hyd. Untersuch." In his article in the "Deutsche Bauzeitung" (No. 82, 1882), more exact values had been obtained by the method of least squares, and these gave

$$v = 0.705\,c + 0.01\,c^2.$$

From an article in the "Deutsche Bauzeitung," it would be seen that another gauging (of the Elbe, above Hamburg) agreed well with this equation; the calculated value was only $2\frac{1}{2}$ per cent. greater than that obtained by measurement. Should this equation prove sufficiently accurate, then the engineer would only need to measure c by surface floats, in order to get a good approximation to the value of v, and from that the discharge.

(II.) *Relation between v and v_s* (p. 38 of "Hyd. Untersuch.")—For this an equation had been formed from seven rivers, or, adding the gauging of the Elbe above Hamburg, from eight rivers. Using the method of least squares to determine the constants, the equation was—

$$v = 0.738\,v_s + 0.05\,v_s^2.$$

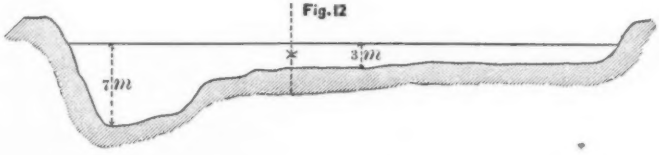
The gauging of the Elbe above Hamburg gave by this equation $v = 1.16$ meter, while by direct measurement $v = 1.17$. However, eight examples for determining the constants were far too few, although the differences were only—

1.10	1.10	0.10 per cent. too great.
2.30	1.00	0.70 " " " small.
		0.00

Further the equation did not apply in cross-sections such as that of the Danube, Fig. 12, given by Harlacher. He presumed the equation would only apply to pretty regular and symmetrical cross-sections. Should the equation be confirmed it would only be necessary to determine the center of figure of a cross-section and measure the velocity at that single point. He wished that both equations might be tried by the Ganges Canal results.

Major Allan Cunningham observed, in reply to the correspondence, that the points noted by Mr. Flamant about comparison of levels of still and running water in free communication, and as to non-existence of surface convexity in permanent régime, and non-existence of transverse surface flow, were of high scientific interest. Further direct experiment was very desirable. As to the mode of weighting the velocity-data of a vertical curve, objected to by Mr. Gordon, it was known that with double-floats the velocity-data increased in accuracy from the bed upwards, so that the weights assigned should also increase from the bed upwards; but at what rate was of course unknown, and therefore a matter of judgment. No stress was laid on the weighting used. Admitting with Mr. Leslie that the action of the air and wind on the projecting portions of the floats used might have exaggerated the observed depression of the maximum velocity-line, still all instruments alike agreed in showing this depression as an existing fact. The irregularity of the bed of the Ganges Canal was not greater than that of most so-called natural streams, as for instance the Thames, in which lumps, bars, hollows, etc., of 1 or 2 feet depth were common: on the vertical scale used in the plates (Vol. II. of the Roorkee Work) these features were enormously exaggerated. Most of the data asked for or suggested as requisite by several of his critics had been actually printed in great detail in Vols. II. and III. of the original

Fig. 12



work. Whilst acknowledging Du Buat's great advance in hydraulics as previously known, it could not be admitted, as said by Mr. Neville, that "nothing better had been done since" in way of a mean-velocity formula. The Darcy-Bazin and the Kutter formulas were both important advances. Also Du Buat's "rational theory," quoted by Mr. Neville, was merely a highly general principle; the detail was still wanting to enable mathematical investigation to be properly applied to the flow of water, and in this sense a "rational theory" was still wanting. If Professor von Wagner's conlcusions, p. 90 (*a*), that the vertical velocity-curve deviated greatly from a parabola near both surface and bed were correct, and that the real bed-velocity was zero, it would be right to give up the use of the parabola approximation. Lastly, Major Cunningham desired to thank the several speakers and writers for the great value of their remarks to him in his special research; his only regret now was that these remarks were not available to help him when the experiments were being made.

PROFESSOR DAVIS ON TERRESTRIAL DEFLECTIONS.

By RD. RANDOLPH, C. E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

IN a review of an article in the February number of VAN NOSTRAND'S MAGAZINE on this subject, Mr. W. M. Davis, of Cambridge, Professor of Geology, who probably relies upon the paper by Farrel, published in the *Cambridge Mathematical Monthly*, 1859, which he refers to, aims to supply two omissions in the article he reviews.

The first of these omissions is that of the consideration of the centrifugal force of the earth which tends to urge a body on its surface, whether in relative motion or rest, outward in the line of a plane radius of the circle of latitude.

While the earth was in a sufficiently fluid state its materials, in obedience to this influence, adjusted themselves into the form of an oblate spheroid and departed from that of the sphere exactly to that degree necessary to bring its attractive and centrifugal forces into equilibrium; and to present a slight gradient, towards the equator, of a perfectly horizontal surface, which is exactly that necessary to counteract the tendency of a detached mass to move on such surface by virtue of the centrifugal force, supposing friction to be absent. Therefore the consideration of this feature was entirely unnecessary in the discussion of the influence of the earth's rotation on derailments. For the hypothesis was that of a perfectly level track; and Prof. Davis admits that it had no influence upon the result. If a hydraulician were calculating the discharge of a fluid through a conduit from a distant elevated source he would omit all considerations of the earth's centrifugal force. He would leave that to those philosophers who maintain that this force is the cause of the flowing of springs.

The second omission is that of the consideration of the law of preservation of areas.

If this view of the subject is contained in the paper by Ferrel in the *Cambridge Mathematical Monthly*, its publication had been better postponed for the twenty-four years that have elapsed. The supposed moving body on the earth's surface is not a projectile nor a satellite moving in an orbit which is the result of the composition of an original projectile force with the attractive force of a primary at the focus. Such a body at every part of its orbit will, in equal periods, pass over arcs which, with the two radii vector including them, will form sectors of equal area, whether the orbit is a circle or the most elongated ellipse. But this is nothing more than the expression of the fact that when the projectile force is in excess, and the centrifugal and centripetal forces not in equilibrium, there will be retardations and accelerations, for the same reasons that the projection and fall of a mass on the earth's surface is retarded and accelerated when gravitation is

not acting at right angles with the path; and that these retardations and accelerations will coincide with the lengthening and shortening of the radius vector; so that the bases and vertical heights of the triangles composing these sectors will vary inversely with a constant ratio; thus determining them to be equal in area. But if any external force were to compel the satellite into a closer orbit without altering its velocity, as would be the case if the force acted for a limited period at right angles with the movement, the areas described in equal periods on the new orbit would still be equal to each other, but they would not be equal to corresponding ones on the old orbit. The result would be a shorter period for an entire revolution, and corresponding areas would be smaller; because one of the factors, the velocity, has not been altered, but the other, the length of the radii vector, have been diminished. So when any detached terrestial mass is moved from one circle of latitude to another, gravitation, acting always at right angles to these circles, will force the mass moving on the surface into a smaller circle of rotation. If it did not encounter friction or other resistance, it would maintain its original velocity and describe an arc of the same length on the smaller circle as that described on the larger circle in the same time. But this arc would be included by shorter radii and the area thus formed would be less; while the area would be greater if the movement was to a circle of greater radius. But the areas described by the cone radii (slant heights) of all these circles from a vertex at the center of a true sphere would be equal in equal periods as a consequence of uniform velocity of rotation. That is to say, that the sum of all the small sectors whose radius is the slant height of the cone, and which are the elements of the cone surface, would be equal in equal periods. Not because there is an arbitrary law for describing equal areas, but because these cone radii are the same at all points of the sphere; and because the velocity of rotation is not altered by the action of gravitation at right angles to any movement over its surface. Therefore Prof. Davis is in error when he assumes that the velocity of rotation is accelerated by moving towards the pole and retarded by moving towards the equator, in obedience to an imaginary law for the preservation of the areas of the circular planes of latitude; and the quantity which he adds to the deflective force is a false quantity. If such a law were in force its maximum effect would be in the case of a movement along a meridian, and would be zero in the case of a movement over a circle of latitude. For in the last case equal areas forbid any increase of velocity of rotation. Yet he has assigned a constant value for all directions and made it equal to that obtained by the considerations of what he calls "the whirling table;" by which, it is to be supposed, that the deflection of the cone radius, represented by the cotangent of latitude, is meant. But it is evident that two aspects of the same fact have been the cause of this confusion.

For the purpose of this dissension the surface of the sphere may be considered as made up of narrow zones of a succession of cones whose slant height is the cotangent of the latitude and whose vertices are at different points in the prolonged axis of the sphere. The difference in the length of any two circles of the same cone is the length of the whole circle which is at the same distance from the vertex as that between the two circles. That part of the cotangent included between the two circles does not describe exactly the same cone as the small one at the vertex only because it is required to suffer translation as well as deflection. The movement of deflection is exactly the same; and which, in one revolution of the sphere, would be the sum of all the small sectors which compose the cone. So that the difference in length of two circles of latitude is the exact measure of the amount of deflection which the cotangent has performed, and which is also measured by the circular base of the small cone at the vertex. If this cone were developed into a plane it would be the sector of a circle showing the whole angle of deflection. Now the part of the cotangent between the two circles of latitude which performs this amount of deflection, is a line belonging to a small horizontal plane which is an element of the surface of the sphere. Consequently this plane is rotating to the same extent that the line is deflecting; just as a wheel is rotating to the same extent as any one of its spokes is deflecting in the plane

of the wheel. Therefore if a body moves along any of these lines, or spokes, it will meet with the horizontal resistance due to the constantly increasing or diminishing circle of rotation of the plane or wheel. Another aspect of the same fact is the passing from one circle of latitude to another and observing the difference in the plane radii of these circles. But the circles are the same whether they be viewed as the bases of cones or the circumferences of circular planes. They measure the deflection of the meridian tangent to the earth's surface and of all the lines in the horizontal plane to which that tangent belongs. And upon whichever of these lines the body may move, north, south, east, or west, the different points of the line, moving with different velocities independent of that of translation, will offer a horizontal resistance to the moving body.

STORAGE BATTERIES.

Report of Prof. HENRY MORTON on the Storage Batteries of the Electrical Power Storage Company, Limited, of London.

HAVING made a series of careful tests and measurements of the electrical storage batteries manufactured by the Electrical Power Storage Company, Limited, at their works at Millwall, London, England, I find the following results in reference to their capacity to store and retain energy, afterwards delivered by them as electric current.

In the first place as to the *capacity* of these batteries in thus *storing* electricity, and its relation to the weight and bulk of such battery or reservoir.

The cells with which I have made my experiments are of the pattern called "one horse power" cells, because they will contain, when fully charged, an amount of energy equal to fully 1,980,000 foot pounds, or one-horse power for one hour.

These cells are externally rectangular wooden boxes, $12\frac{1}{2}$ inches high, $11\frac{1}{2}$ inches wide and $5\frac{3}{4}$ inches thick.

Two of them side by side, as they would stand when in use, occupy about one cubic foot of space.

Each cell contains 16 plates, whose united weight is 48 pounds, and with the lead-lined box and liquid, the entire weight of the cell when in use, is $79\frac{1}{2}$, or say 80 pounds.

One of these cells fully charged will yield, as I have found by careful experiment, a current of 32.5 amperes at the beginning, and 31.2 amperes at the close of a continuous discharge for nine hours.

This amounts to 286.5 ampere hours of current, and if even short interruptions or periods of repose occur in the use of the current a yet larger total amount can be obtained.

An Edison incandescent lamp of high resistance, giving a light of 16 candles, requires a current of .73 of an ampere to supply it. Such a current, therefore, as these batteries yield for nine hours at a time, will suffice for 44 such lamps.

To secure sufficient electro-motive force or propelling power to overcome the resistance of these lamps would, however, require about 50 of such cells, so that a battery of 50 of these cells connected in series, would operate 44 lamps for nine hours, or for even a longer time in the aggregate, if the use were interrupted, as it would be in practice.

If fewer lamps were used with the same battery, they would be operated for a proportionately longer time.

Thus 11 lamps would be supplied by a 50 cell battery for thirty-six hours of continuous action; or as lights are commonly used in private houses on the average for five hours each night, such a battery once charged, would operate 11 lamps for a week.

To express the relation between weight of battery and power of maintaining a light we might, therefore, say that for each lamp operated for nine hours 1¼ cells of battery would be required, or a weight of about 90 pounds of battery. This would be for each hour of burning each lamp, 10 pounds of battery.

This makes a very simple rule for calculating the weight of battery required for any number of lamps for any time.

Thus, suppose we wish a battery to

operate 25 lamps for five hours each night, the battery being recharged during the day. We have 25x5x10=1,250 pounds as the weight of battery required.

Comparing the efficiency of this storage battery with that of other similar arrangements, such as the Faure battery, measured and reported upon by M. Tresca, of the "Conservatoire des Arts et Metiers," it shows a marked superiority.

Thus in M. Tresca's experiments a cell weighing 95 pounds, yielded a current representing 793,791 foot-pounds of energy where your battery yields, as I have stated above, 1,826,168 foot-pounds and only weighs 80 pounds.

Your battery, therefore, yields more than twice the energy and weighs almost one-fifth less.

Even the experiments made some time afterwards by Professors Ayrton and Perry, on other Faure accumulators, though the conditions were rendered as favorable as possible by distributing the discharge over three periods of six hours each on three successive days, do not show a much better result. In this case the weight of the battery *plates* only is given, and that is 81 pounds. Reducing the results proportionally for batteries whose plates weigh 48 pounds, I find that by the experiments of Professors Ayrton and Perry each cell of this weight should give 853,333 foot-pounds of energy.

This again is less than half the amount of energy which I have repeatedly obtained from your battery.

Passing next to the efficiency of your batteries as regards their delivery of nearly the same current as was used to charge them, I have found that the loss in this relation is less than 10 per cent. In other words, in my experiments I have obtained from these batteries 90 to 91 per cent. of the current used to charge them. This far exceeds the results obtained by M. Tresca with the Faure batteries.

Tresca reports that he recovered only 60 per cent. of the current used to charge the battery, and Ayrton and Perry found the loss in charging and discharging to be "not greater than 18 per cent."

In this, however, is included the loss due to the fall of electro-motive force in the discharge as compared with the charge. This I find to be on the average less than $\frac{2}{10}$ of a volt, which would bring the total loss of available energy obtained from the battery as compared with that expended in charging it to 18 per cent.

Lastly comes the very important question as to the retention of charge during a long time. To test this I charged three cells and locked them in a closet on February 1st, where they remained until February 16th, when I began discharging them at the rate of 32 amperes, continuing this rate of discharge on the next day. I thus obtained 266.7 ampere hours of current.

Comparing this with the 286.5 ampere hours of current obtained from the other cells which I discharged soon after charging them, shows a loss of 7 per cent. caused by standing for 16 days.

The above measurements and comparisons show that this storage battery has attained a degree of efficiency which will render it applicable to a number of uses.

Thus, for example, on steam boats, by the use of such storage batteries, the irregular and occasionally interrupted motion of the main engine might operate a relatively small dynamo-electro machine so as to charge the batteries during the entire twenty-four hours, and the current from these batteries would then supply light with perfect steadiness during the relatively brief time in which it is required. In this way the cost of supplying and running a special and large engine, which would be needed for operating the same lights directly without the storage battery, would be avoided, and also the necessity for extreme steadiness in running the dynamo, and all risks of extinction of the lights from a mometary interruption of motion in any part of the machinery would be removed, as the battery would secure an absolutely steady and continuous supply of current no matter how little regular might be the action of the engine or dynamo-electric machine.

Again, in larger buildings the engine used to operate the elevator or to do any other work, if of sufficient power could charge the storage battery without interrupting its regular work, and thus

supply the light needed at a minimum cost for special machinery and skilled supervision.

In private houses where the running of a large engine with extreme smoothness and absolute certainty during the hours when light is needed, would be out of the question, a small engine operated at convenient intervals, and with no need of regularity or of fixed hours, would accomplish all that was required if a storage battery was employed with it.

I am informed that these storage batteries have been successfully applied to the running of street cars, and in view of the fact that they will contain almost 2,000,000 foot-pounds of energy for each 80 pounds of dead weight, I should consider the prospect in this direction most encouraging.

In short it seems to me that there is manifestly a wide field for the profitable employment of this greatly improved form of the electric storage battery.

DISTRIBUTION OF WATER FROM IRRIGATION CANALS.

By M. BRICKA.

From "Annales des Ponts et Chaussées," for Abstracts of the Institution of Civil Engineers.

THIS article indicates the importance of a careful system for the distribution of water for irrigation, as gathered from the experience of the Verdon Canal, a matter often overlooked in the construction of irrigation works.

Irrigation in Provence not merely increases the return from the land, but also enables any crop to be cultivated where failing a supply of water, wheat, olives, and almonds are almost the only produce. The gross return, which formerly hardly exceeded from £6 10*s.* to £8 per acre, has been raised by irrigation to from £13 to £24 per acre, in meadow land, and from £16 to £32 for potatoes, affording a very large increase in the net profit, which was formerly only about £1 12*s.* an acre. The profit in small holdings, from the culture of fruit and vegetable, is still larger. The water is generally supplied from April to September, over the whole surface for grass crops, and in channels for fruit and vegetables. It has been found in practice that a supply equivalent to $5\frac{1}{3}$ gallons per minute per acre is sufficient; it is distributed periodically, the interval between the commencement of two successive waterings having been fixed for the distributing channels of the Verdon Canal at six days and a quarter, so that each proprietor may have his fair turn of day and night supervision. The rate of discharge which has been found suitable, so that, without undue expenditure of time in delivery, the supply may not exceed what can be properly distributed by the proprietor, is between 400 and 530 gallons per minute, and the quantity supplied to different properties is regulated by a variation, not of the rate of discharge, but of the period of flow.

The Verdon Canal has been previously described; and the author confines himself to a description of the distributing channels which bring the water to the head of the estate of each subscriber, and which were constructed by the same company who had the concession for the Verdon Canal. There are about 15 miles of these channels to every 1,000 acres. As economy is essential in forming the distributing channels, they have to follow the surface of the country, and to avoid severance; and the constructive works have to be restricted to falls, and crossings of unimportant roads and streams. The channels are accordingly very irregular, both in direction and slope; and numerous falls connect abrupt changes of level. Where a hard stratum exists near the surface, the amount of slope given to the channel is immaterial; but in a bed liable to scour, the rate of flow must not exceed from 2 feet 3 inches to 2 feet 8 inches per second. The flow, however, must not be less than 1 foot per second, otherwise the silt, which the water brings down in abundance after rain, or from the melting snows, would deposit and obstruct the channel. In places where it is inconvenient to replace a quick slope by a fall, scour has been prevented by dry rough pitching, resting upon a layer

of broken stone, which impedes the undermining action of the current below the pitching. The section of the channels is generally flat-bottomed, with side slopes of 1 to 1; but sometimes through firm ground, and where scour is needed, the section is made rectangular. The works of construction, which are very numerous, consist of modifications, according to circumstances, of simple types, of which various specimens are given in the plates. The total cost of the 250 miles of distributing channels, in the irrigated area of 18,530 acres in the Aix district, was about £29,600, or 1*s.* 4½*d.* per lineal yard.

The author enters fully into the methods by which the construction and control of the distributing channels can be arranged, and shows the obstacles in the way of entrusting these matters to an association of the proprietors. If the State does not undertake it, the only other suitable authority is a company with a concession. It is impossible to estimate at the outset what the demand for water will amount to. Some soils, though apparently very dry, have a clay subsoil which retains moisture sufficient for good crops of wheat, and irrigation would only render the ground marshy. Small proprietors use a large quantity of water, as they irrigate all their land; whilst large proprietors rarely irrigate more than a third. Also the change of crops takes place slowly, so that at the commencement the demand for water is comparatively small. It is accordingly advisable not to extend the distributing channels till an adequate number of subscribers come forward. It is very important that the supply shall be delivered with great regularity, to prevent injury to the crops depending on it, and that no waste should occur. This is most effectually secured by entrusting the management of the sluice gates to responsible persons, not interested in the irrigation, who distribute the water according to regulations arranged each year, and who also are on the watch to prevent fraudulent misappropriation of the supply. Under these circumstances, the cost of allotment in a year amounts roughly to between 29*s.* and 47*s.* per gallon of water distributed per second; and the cost of maintenance, exclusive of large repairs, may vary from 20*s.* to 34*s.* for distributing channels situated in conditions similar to those of the Verdon canal.

ON THE ADJUSTMENT OF CONDITION-OBSERVATIONS IN THE METHOD OF LEAST SQUARES, WITH ITS APPLICATION TO GEODETIC WORK, AND WITH SPECIAL REFERENCE TO AMERICAN METHODS OF THE ADJUSTMENT OF A SYSTEM OF TRIANGULATION.

By T. W. WRIGHT, B. A., C. E., Lehigh University, Bethlehem, Pa.
Late Assistant Engineer U. S. Lake Survey.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

II.

40. *The Side-Equations.*

In a single triangle, or in a simple chain of triangles, we have seen that the length of any assigned side can be computed from a given side in but one way. The angle-equation is the only condition-equation. When the triangles are interlaced this is not so.

Thus in the quadrilateral ABCD, all four stations being occupied, the three equations given by the triangles DAB, ABC, BCD, that is:

$$\begin{aligned} D_1AB + ABD_1 + BD_1A &= 180 + \varepsilon_1 \\ ABC + BCA + CAB &= 180 + \varepsilon_2 \\ BCD_2 + CD_2B + D_2BC &= 180 + \varepsilon_3 \end{aligned}$$

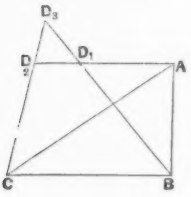
may be satisfied, and yet the figure not be a perfect quadrilateral. The further condition that the lines CD_2D_2, BD_1D_2, AD_1D_2 intersect in the same point, that is, that the computed values of the lines BD_1, BD_2 are equal, must be added.

Proceeding from a base (as AB) we may compute BD_1 directly from the triangle

D_1AB and BD_2 through the triangles ABC, BCD_2. This gives us

$$\frac{\sin.BD_1}{\sin.BA}=\frac{\sin.BAD_1}{\sin.AD_1B},$$

$$\frac{\sin.BD_2}{\sin.BA}=\frac{\sin.DCB}{\sin.ACB}\cdot\frac{\sin.BAC}{\sin.BDC}.$$



But BD_1 must equal BD_2.

Hence the condition

$$\frac{\sin.BAD}{\sin.ADB}\cdot\frac{\sin.ACB}{\sin.BAC}\cdot\frac{\sin.BDC}{\sin.DCB}=1.$$

This condition-equation is called a side-equation or sine-equation, and the vertex B is called the *pole* of the quadrilateral for this equation.

41. The side-equation, deduced for spherical triangles, must hold in most cases that occur in practice for the corresponding plane triangles, the angles of each triangle being adjusted according to Legendre's theorem. We have then

$$\frac{BD_1}{BA}=\frac{\sin.(BAD_1-\varepsilon_1)}{\sin.(BD_1A-\varepsilon_1)},$$

$$\frac{BD_2}{BA}=\frac{\sin.(BCD-\varepsilon_2)}{\sin.(BDC-\varepsilon_2)}\cdot\frac{\sin.(BAC-\varepsilon_2)}{\sin.(BCA-\varepsilon_2)}$$

But $BD_1=BD_2$,

and therefore

$$\frac{\sin.(BAD-\varepsilon_1)}{\sin.(BDA-\varepsilon_1)}\cdot\frac{\sin.(ACB-\varepsilon_2)}{\sin.(BAC-\varepsilon_2)}\cdot\frac{\sin.(BDC-\varepsilon_2)}{\sin.(ACD-\varepsilon_2)}=1$$

where ε_1, ε_2, ε_3, denote one third of the spherical excesses of the triangles BAD, BAC, and BCD, respectively.

As the two forms lead to the same result it affords a check of the accuracy of the work to compute the side-equation in both ways. It is evidently more simple to use the spherical angles than the plane angles.

42. In a spherical triangle, the sines of the angles being proportional to the sines of the opposite sides, the side-equation

$$\frac{\sin.BAD}{\sin.BDA}\cdot\frac{\sin.BCA}{\sin.BAC}\cdot\frac{\sin.BDC}{\sin.BCD}=1$$

if satisfied, gives the identical relation

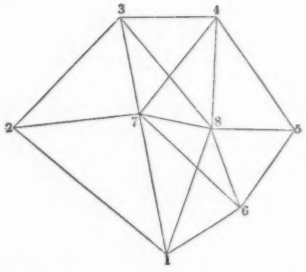
$$\frac{\sin.BD}{\sin.BA}\cdot\frac{\sin.BA}{\sin.BC}\cdot\frac{\sin.BC}{\sin.BD}=1.$$

Hence in writing down a side-equation having chosen the pole (B for example) it is convenient to write down first the scheme

$$\frac{BD}{BA}\cdot\frac{BA}{BC}\cdot\frac{BC}{BD}=1,$$

formed by the lines radiating from the point B and then the sine-equation follows at once.

43. If the triangulation-net instead of involving quadrilaterals only involves central polygons such that we can pass from one side to any other through a chain of triangles, the same process is followed in forming the side-equations as in a



quadrilateral. Thus in the figure which represents part of the triangulation of Lake Erie west of Buffalo Base, there are from side-equations,

the quadrilaterals, 7348, 1786,
the pentagons, 71238, 87456.

The scheme for the pentagonal side-equation 71238 for example would be just as in the case of a central quadrilateral, taking 7 as pole,

$$\frac{71}{72}\cdot\frac{72}{73}\cdot\frac{73}{78}\cdot\frac{78}{71}=1,$$

and hence

$$\frac{\sin.721}{\sin.712}\cdot\frac{\sin.732}{\sin.723}\cdot\frac{\sin.783}{\sin.738}\cdot\frac{\sin.718}{\sin.781}=1.$$

44. Thus far we have considered the side-equations in their rigorous form. But in order to treat them by the method of least squares they must first be reduced to the linear form which may be done as follows:

Let the side-equation be

$$\frac{\sin.A_1.\sin.A_3\ldots.}{\sin.A_2.\sin.A_4\ldots.}=1,$$

where $A_1, A_2\ldots$ denote the most probable values of the angles. Let $M_1, M_2, \ldots.$ denote the measured values and $v_1, v_2, v_3\ldots$ the most probable corrections to these values, which corrections are assumed to be small. Then the equation may be written

$$\frac{\sin.(M_1+v_1).\sin.(M_3+v_3)\ldots.}{\sin.(M_2+v_2).\sin.(M_4+v_4)\ldots.}=1.$$

Taking the log. of each side of this equation and expanding by Taylor's theorem we have, taking the first powers of the corrections only,

$$\log.\sin.M_1+\frac{d}{dM_1}(\log.\sin.M_1)v_1-$$
$$\left\{\log.\sin.M_2+\frac{d}{dM_2}(\log.\sin.M_2)v_2\right\}$$
$$+\ldots..=0 \quad (81).$$

which may be written in two forms for computation.

First if the corrections to the angles are expressed in seconds we may put

$$\frac{d}{dM_1}(\log.\sin.M_1)=\delta',$$

when δ' is the tabular difference for $1''$ for the angle M_1 in a table of log. sines. Then we have

$$\delta'v_1-\delta''v_2+\ldots+\log.\sin.M_1-\log.\sin.M_2+\ldots=0$$

that is $$[\delta v]=l \quad (82).$$

where l is a known quantity.

Secondly and specially for small angles and angles near 180° we may replace

$$\frac{d}{dM_1}(\log.\sin.M_1)$$ by (mod.) sin. $1''$ cot. M_1

when (mod.) denotes the modulus of the common system of logarithms. Equation (81) may then be arranged

$$\cot.M_1.v_1-\cot.M_2.v_2+\ldots$$
$$\frac{1}{(\text{mod.})\sin.1''}(\log.\sin.M_2-\log.\sin.M_1+\ldots)$$
$$\log.\frac{1}{10^7(\text{mod.})\sin.1''}=8.67664-10.$$

For convenience of computation there is not much to choose between the two forms. The second is perhaps on the whole to be preferred.

Example.—The quadrilateral N. Base, S. Base, Lester, Oneota.

Take the pole at Lester.

We have the scheme

$$\frac{LS}{LN}\cdot\frac{LN}{LO}\cdot\frac{LO}{LS}=1,$$

and the sine-equation.

$$\frac{\sin.(M_2+v_2)}{\sin.(M_3+v_3)}\cdot\frac{\sin.(M_3+v_3)}{\sin.(M_1+v_1)}\cdot$$
$$\frac{\sin.(M_6+v_6)}{\sin.(M_7+M_6+v_7+v_6}=1.$$

First form,

	log. sin.
113 39 05.07+v_2	=9.9618969.7− 9.22 v_2
43 46 26.40+v_7	=9.8399903.7+21.98 v_7
70 39 24.60+v_6	=9.9747656.9+ 7.39 v_6
	530.0
47 31 20.41+v_5	=9.8677859.5+19.28 v_5
124 09 40.69+v_1	=9.9177470.2−14.29 v_1
78 27 06.06+v_7+v_6	=9.9911180.3+4.30(v_7+v_6)
	510.0
	+20.0

Check by deducting $\frac{1}{3}$ of the spherical excess of the triangle to which it belongs.

Angle.	log. sin.		
° ′ ″		° ″ ″	
113 39 05.00	9.9618970.3	47 31 20.34	9.8677858.2
43 46 26.36	9.8399902.5	124 09 40.65	9.9177470.7
70 39 24 48	9.9747656.0	78 27 05 93	9.9911179.8
	528.8		508.7
	+20.1		

agreeing closely with the value found from the spherical angles. Hence the side-equation is

$$14.29v_1-9.22v_2-19.28v_3+7.39v_4+$$
$$17.68v_7-4.30v_6+20.0=0$$

or dividing by 10 which is approximately the mean of the coefficients and which amounts to the same thing as expressing the log. differences in units of the sixth place of decimals,

$$1.43v_1-0.92v_2-1.93v_3+0.74v_4+1.77v_7-0.43v_8+2.00=0.$$

Second form.

log. sine.	cot.	log. sine.	cot.
9.9618969.7	$-0.4380v_2$	9.867859.5	$+0.9156v_3$
9.8399903.4	$+1.0437v_7$	9.9177470.2	$-0\ 6786v_1$
9.9747656.9	$+0.3510v_4$	9 9911180.3	$+0.2043(v_7+v_8$
530.0		510.0	
510.0			
20.0 log.	1.30103		
	8.67664		
	9.97767	0.9499	

and the side-equation is

$$+0.6786v_1-0.4380v_2-0.9156v_3+0.3510v_4+0.8394v_7-0.2043v_8+0.9499=0$$

Each term of this equation is of course in a constant ratio to the corresponding term of the previous form.

It will be noticed that the log. sines have been carried out to eight places of logs. only. On the Coast Survey and Lake Survey the practice is to carry out the log. sines to ten places of decimals, but on some of the leading European Surveys, such as the Prussian, Danish and Spanish, the log. sines are carried out to only eight places. On the English Ordnance Survey they were carried out to ten places, and on the Great Trigonometrical Survey of India to seven places.

On the Coast Survey the log. differences (δ) for 1″ are computed in units of the fifth place of decimals, on the Lake Survey in units of the seventh decimal place, and then the side-equation was divided by a such a number as would make the average of the coefficients equal unity approximately. By others the sixth place of decimals is taken as the unit for the final values of the coefficients. This I should in general prefer as being most convenient. Or still better, since in a good system of triangulation the angles in the side-equations should range between 30° and 60° and the difference for log. sin. 45° is 21, we may choose 20 as on the whole the best of any single divisor for differences expressed in units of the seventh place of decimals.

The principle involved in all of the preceding is that since the coefficients of the corrections in the angle equations are +1 or −1, and these are absolute, the greatest possible equality of coefficients throughout the condition equations is best in order to secure an equable distribution of errors arising from imperfect computation.

A striking difference between condition equations and observation-equations is here brought out. A condition-equation expressing a rigorous relation among the quantities independent of the observations may be multiplied or divided by any number without affecting that relation: with an observation-equation, on the other hand, the effect would be to increase or diminish its weight.

The coefficients of the corrections are usually carried out one place further than the absolute term in a side equation. This for a short computation would be unnecessary, but in a long one, on account of cutting off the last figures in products and quotients all through, errors accumulate so that we must carry with us more decimal places than we finally wish to keep.

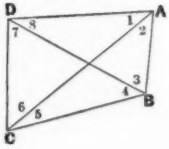
45.—*Number of side-equations in a quadrilateral.*

As any one of the six lines in a quadrilateral may be assumed as base there may be *

$$\frac{6\times5}{1\times2}=15$$

side-equations.

Thus in the quadrilateral ABCD considering AB as base we have for computing AC, AD, or taking AC as base and computing AD the same equations,



$$\frac{\sin.AC}{\sin.AB}=\frac{\sin.(3+4)}{\sin.5},\quad \frac{\sin.AD}{\sin.AC}=\frac{\sin.6}{\sin.(7+8)},$$

$$\frac{\sin.AB}{\sin.AD}=\frac{\sin.8}{\sin.3},$$

and ∴

$$\frac{\sin.(3+4)\sin.6.\sin.8}{\sin.5.\sin.(7+8)\sin.3}=1.$$

* See Zachariæ geodätische Hauptpunkte. Zeitschr. für Vermess. Vol. IX.

This form with A as pole which results from the scheme

$$\frac{AC}{AB}\cdot\frac{AD}{AC}\cdot\frac{AB}{AD}=1,$$

we shall call (A).

The equations corresponding to the poles B, C, D, are:

$$\frac{\sin.(5+6)}{\sin.7}\cdot\frac{\sin.8}{\sin.(1+2)}\cdot\frac{\sin.2}{\sin.5}=1 \qquad (B)$$

$$\frac{\sin.(7+8)}{\sin.7}\cdot\frac{\sin.2}{\sin.(3+4)}\cdot\frac{\sin.4}{\sin.7}=1 \qquad (C)$$

$$\frac{\sin.(1+2)}{\sin.3}\cdot\frac{\sin.4}{\sin.(5+6)}\cdot\frac{\sin.6}{\sin.1}=1 \qquad (D)$$

Since $(A)\times(C)=(B)\times(D)$ there are only three *independent* forms resulting from the first twelve forms, though there are four that are *different*.

Again computing BC from AD through the diagonals, the side-equation is

$$\frac{\sin.2}{\sin.6}\cdot\frac{\sin.3}{\sin.7}\cdot\frac{\sin.(5+6)}{\sin.(3+4)}\cdot\frac{\sin.(7+8)}{\sin.(1+2)}=1$$

which is equivalent to

$$(B)\div(A) \text{ or } (C)\div(D)$$

and it gives no new relation.

Computing CD from AB through the diagonals the equation is

$$\frac{\sin.1}{\sin.4}\cdot\frac{\sin.8}{\sin.5}\cdot\frac{\sin.3+4}{\sin.1+2}\cdot\frac{\sin.5+6}{\sin.7+8}=1$$

which is equivalent to

$$(A)\div(D) \text{ or to } (B)\div(C).$$

Computing the side BC from AD or CD from AB though the sides gives the equation

$$\frac{\sin.1}{\sin.2}\cdot\frac{\sin.3}{\sin.4}\cdot\frac{\sin.5}{\sin.6}\cdot\frac{\sin.7}{\sin.8}=1$$

which is equivalent to

$$(A)\times(C) \text{ or } (B)\times(D)$$

The seven forms then expressed in terms of the first three are

$$(A),\ (B),\ (C),\ \frac{(A)\,(C)}{(B)},\ (A).(C),\ \frac{(B)}{(C)},\ \frac{(B)}{(A)}.$$

We have thus seven different forms, though only three that are independent out of the fifteen.

If we take the angle-equations into account, a little consideration will show that these three forms are really one.

For equation (A) means that the points D_1, D_2 coincide. (See Fig. Art. 40.) Equation (B) that D_1, D_3 coincide, and equation (C) that D_2, D_3 coincide. If any one of these conditions is satisfied so are all the others, as all amount to the same condition, that D is not three points, but one point.

The result is that a quadrilateral is determined by three angle-equations and one side-equation. This may be expressed in 56 ways, as each of the 8 sets of angle equations may be combined with each of the 7 side-equations.

46. Taking the seven different forms of the side-equation in a quadrilateral, the conclusion was reached in (45) that any one of these forms was as good as any other in satisfying the conditions imposed. But now that the side equations are no longer rigorous, but only approximate, the question may be asked, Is not some one of the seven to be preferred? This we proceed to examine.

Now we have assumed in reducing the side-equations to the linear form, that the angles in these equations are such that terms of the expansions of the log. sines of these angles involving powers of the corrections above the first, may be neglected. This places the linear equations on the same footing as the rigorous equations.

On the other hand, we must consider that the corrections arising from a side-equation are distributed as the cotangents of the angles entering it. If only the five angles necessary for forming a side-equa- were measured, there would be no angle equations, and the corrections arising from the side-equation would be the more irregularly distributed the more irregular the size of the angles. When all the angles are measured, the modifying influence of the angle-equations is such as to balance this irregularity. Hence on this account the importance of measuring all the angles necessary for forming the complete series of condition-equations.

In a central system involving pentagons, hexagons, &c., by taking the pole at the center, the resulting side-equations follow the law of the angle equations nearly. When well hinged together this system is to be preferred, but is much more costly. In the United States the system of quadrilaterals has been followed.

In fact, since a side-equation expresses

but the computation of one side from another by different routes, and we know that the best-shaped triangles should be chosen, it follows that in laying out the work, the angles formed by the diagonals should be taken into consideration so that all small angles be avoided.

There are, however, so many practical difficulties in the way that theoretical considerations of this kind have often to be set aside in part at least.

47. The danger of introducing very small angles in a side-equation may be illustrated by the quadrilateral Farquhar, Outer Island, W. Sawteeth, E. Sawteeth in the triangulation of Lake Superior.

Given, the angles all of the same weight *

	°	′	″	
EFW	0	19	12.80	(1)
WFO	37	41	15.42	(2)
FOE	77	48	12.97	(3)
EOW	2	12	20.89	(4)
OWF	62	18	23.662	(5)
FWE	12	52	20.308	(6)
OEF	64	11	34.564	(8)

The spherical excesses of the triangles FWO, FEO are 15″.845, 15″.577 respectively. The angle-equations are

$$v_2+v_3+v_4+v_5=2.903$$
$$v_1+v_2+v_3+v_5=0.177$$

Taking the pole at O the side-equation is

$$-2.694\,v_1+0.031\,v_2-0.471\,v_4-1.019\,v_5+0.086\,v_6+1.018\,v_5=-2.2502.$$

* Report of Chief of Engineers, 1873.

Taking the pole at E the side-equation is

$$+5.343\,v_1-0.038\,v_2+0.006\,v_3-0.781\,v_4-0.008\,v_5+0.124\,v_6=-1.2533.$$

The corrected angles resulting from the two solutions are

°	′	″	°	′	″
0	19	12.718	0	19	12.557
37	41	15.768	37	41	15.805
77	48	13.325	77	48	13.359
2	12	21.937	2	12	21.920
62	18	24.815	62	18	24.760
12	52	20.291	12	52	20.297
64	11	33.766	64	11	33.855

Taking the side OW as base (=1) let us compute the sides from the different triangles possible, and from the values found by the two adjustments in succession.

Triangle.	Angle, adjusted.	Ang'e, plane.	log. sin.	log. side.
EOW	° ′ ″	″		
	102 36 53.381	53.240	9.9893877.1	0
	2 12 21.937	21.796	8.5853862.8	8.5959985.7
	75 10 45.106	44.964	9.9853053.7	9.9959176.6
EOF				
	38 00 28.486	28.294	9.7894047.5	9.9959176.6
	64 11 33.766	28.573	9.9543643.1	0.1608772.2
	77 48 13.325	08.133	9.9900831.0	0.1965960.1

SECOND ROUTE.

OWF				
	37 41 15.768	10.486	9.7862807.4	0.
	62 18 24.815	19.534	9.9471579.8	0 1608772.4
	80 00 35.262	29.980	9.9933625.8	0.2070818.4
EFW				
	166 48 27.147	27.095	9.3583593.7	0.2070818.4
	0 19 12.718	12.666	7.7472760.9	8.5959985.6
	12 52 20.291	20.239	9.3478735.5	0.1965960.2

The agreement of the sides is here very satisfactory.

From the values of the angles found by taking the pole at E, we find passing through the same triangles.

Triangle.	Angle, adjusted.	Angle, plane.	log. sin.	log. side.
EOW	° ′ ″	″		
	102 36 53.447	53.306	9.9893876.7	0.
	2 12 21.920	21.779	8.5853853.5	8.5959976 8
	75 10 45.057	44.915	9.9853053.5	9.9959176.8
EOF				
	38 00 28.362	28.170	9.7894044.1	9.9959176.8
	64 11 33.855	28.663	9.9543644.0	0.1608776.7
	77 48 13.359	08.167	9.9900831.2	0.1965963.9

SECOND ROUTE.

OWF				
	37 41 15.805	10.524	9 7862808.5	0.
	62 18 24 760	19.478	9.9471579.2	0.1608770.7
	80 00 35.279	29.998	9 9933625.9	0.2070817.4
EFW				
	166 48 27.302	27.250	9 3583579.8	0.2070817.4
	0 19 12 557	12.505	7.7472154.3	8.5959391.9
	12 51 20.297	20.245	9.3478736.1	0.1965973.7

The agreement here especially for the side EW is quite unsatisfactory.

48. We are now in a position to estimate the *number of independent side-equations in any triangulation-net*

The base line being measured its extremities are known. To fix a third point two directions must be measured; in other words, we must know the other two sides of the triangle of which this point is the vertex. Each superfluous line will therefore give a side-equation. Hence, if we have a net of triangles connecting s stations, two of the stations being the ends of the base, we must have in order to plot the figure $2(s-2)$ lines besides the base. If, then, the total number of lines in the figure is l, the number of superfluous ones must be

$$l-[2(s-2)+1]$$

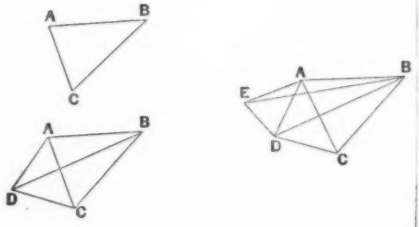
giving in all

$$l-2s+3 \qquad (83)$$

side-equations.

50. ***How to pick out the angle and side-equations.***

In the selection of the side and angle-equations in a triangulation-net we have two dangers to guard against: first, that we omit no necessary conditions, and second, that we admit no unnecessary ones. The rule usually followed (due to Bessel) is to start from some line, usually a base-line, and add point to point, writing down the conditions that express the connections of each point to the system as the system grows.



For example, take AB as base. A third point C, all the angles being measured gives an angle-equation from the triangle ABC.

A fourth point D gives in addition

Angle-equations from ABD, ACD,
Side-equation from ABCD.

A fifth point E gives in addition

Angle-equations from ABE, AED.
Side-equation from ABED,

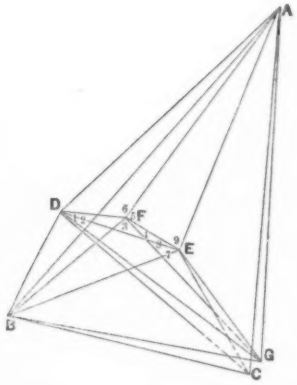
and so on.

We have then in one complete figure five angle-equations and two side-equations.

Check—No. of stations = 5.
No. of lines = 9.

$\therefore$ No. of angle-equations $=9-5+1=5$
and No. of side-equations $=9-10+3=2$.

51. As an illustration of the difficulties which may arise in solving a triangulation-net, I shall take the triangulation around the Chicago Base. It is represented in the figure.



The peculiarity of this system is that the station F is nearly on the line DE. Thus the triangle equation DEF

$$\begin{aligned}
\sphericalangle \text{EDF} &= 00\ \ 00\ \ 00.976+v_1\\
\sphericalangle \text{FED} &= 00\ \ 00\ \ 1.250+v_2\\
\sphericalangle \text{DFE} &= 179\ \ 59\ \ 56.897+v_3\\
&\quad 179\ \ 59\ \ 59.123+v_1+v_2+v_3\\
&\quad 180\ \ 00\ \ 00.000\\
0 &= -0.877+v_1+v_2+v_3.
\end{aligned}$$

From the rules given above it follows that there are ten angle equations and eight side-equations in the system.

In the selection of the side-equations it is advisable to avoid those quadrilatera lsentangled with the above triangle. They are ADEF, BDEF, GDEF.

For example, if we take the quadrilateral GDEF we have in units of the seventh place of decimals, pole at G,

$$54.7729v_1-0.0008v_2+0.0015v_4-40.7066(v_5+v_6)+26.1803v_7-0.0003(v_8+v_9)-40.130=0.$$

Now, since the co-efficients of v_2, v_4, v_5, v_9, are each less than 2 in the tenth place of decimals, the above equation is nearly the same as

$$5.4773v_1-4.0707(v_5+v_6)+2.6180v_7-4.0130=0.$$

or dividing by 4 and replacing v_5+v_6 by $-v_3$, which we can do since the sum of the angles at F is 360.

$$1.3693v_1+1.0177v_3+0.6545v_7-1.0032=0$$

which is nearly the same as the above angle-equation, and therefore can scarcely be said to be a side-equation at all.

Similarly the quadrilaterals ADEF, BDFF, give respectively, neglecting co-efficients less than 5 in the fifth place of decimals.

$$1.2191v_1+0.2085v_7+0.7220v_9-0.7899=0$$
$$0.2560v_1+2.4665v_7+1.3434v_9-0.8355=0$$

both of which merely express approximately the same relations among the angles of the small triangle DEF as the angle-equation formed from that triangle. We therefore conclude that in the formation of the angle and side-equations the triangle DEF should be avoided.

52. There is another trouble caused by the introduction of relatively small angles in side-equations, and where consequently the coefficients of some of the unknowns greatly exceed in size some of the others. It is this: in the normal equations the side terms may be as large as, and even larger, than the diagonal term. In our example above, some of the side terms would exceed the diagonal terms if worked with the side-equations mentioned. As the diagonal terms are the divisors in the Gaussian method of solution of the normal equations, they ought to be larger than the terms into which they are divided, to obtain a closely accurate result, especially if any approximate method of elimination is employed. Of course the solution can be carried through, but the number of decimal places to which it must be carried to make the values obtained of any worth, causes a great amount of labor which might have been saved.

Thus the terms of the correlate normal equations found with the side-equations of Art. 51 are as follows, in the neighborhood of the diagonal term:

$$\begin{array}{l} 7.44+\ 7.12-\ 9.33\ +\ldots\ldots\ldots \\ 7.12+28.62-12.70\ +\ldots\ldots\ldots \\ -9.33-12.70+12.34\ +\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots \end{array}$$

From the equations no exact values could be found, without an excessive amount of computation.

53. Again in selecting the side-equations in a net, care must be taken that only independent conditions are chosen. Thus we might have chosen for the eight quadrilaterals the following:

AGBD, AGDE, AGBE, BDFG, BFEG, AGDF, ACBE, BDEG.

But a careful examination will show that these are not independent. For taking the four AGBD, AGDE, AGBE, BDEG, and choosing G as pole, which is common to them all, we have

$$1=\frac{\sin.\,ADG}{\sin.\,DAG}\cdot\frac{\sin.\,DBG}{\sin.\,BDG}\cdot\frac{\sin.\,BAG}{\sin.\,ABG}$$
$$1=\frac{\sin.\,DAG}{\sin.\,ADG}\cdot\frac{\sin.\,EDG}{\sin.\,DEG}\cdot\frac{\sin.\,AEG}{\sin.\,EAG}$$
$$1=\frac{\sin.\,ABG}{\sin.\,BAG}\cdot\frac{\sin.\,BEG}{\sin.\,EBG}\cdot\frac{\sin.\,EAG}{\sin.\,AEG}$$
$$1=\frac{\sin.\,BDG}{\sin.\,DBG}\cdot\frac{\sin.\,DEG}{\sin.\,EDG}\cdot\frac{\sin.\,EBG}{\sin.\,BEG}$$

which equations multiplied together reduce to the identical form

$$1=1,$$

showing that from any three the fourth may be found.

If the rule given in (50) is followed this repetition of conditions will hardly occur.

The entrance of mutually dependent conditions would, however, be detected in the course of the elimination as we should arrive at two identical equations, or in other words one of the correlates would become indeterminate. If this should happen in the course of a solution we should at once conclude that the trouble was in the condition-equations and proceed to re-examine them.

54. We next pass on to the *arrangement* of the condition-equations in the net adjustment. On first thoughts it might seem that it would be well to arrange the angle-equations and side-equations in two separate sets and so carry

them forward for solution. This was done in some of the older work. The only objection to doing it is that the process of finding the corrections is more troublesome. Experience shows that the solution of a series of normal equations is much facilitated if the coefficients are arranged as the steps of a stair rather than irregularly. Thus,

$$\begin{aligned} [aa]x+[ab]y &=[al] \\ [bl]y+[bc]z &=[bl] \qquad 1 \\ [bc]y+[cc]z &=[cl] \end{aligned}$$

is a more convenient form for solution than

$$\begin{aligned} [aa]x+\quad [ab]y &=[al] \\ [bc]z+[bc]y &=[bl] \qquad 2 \\ [ab]x+[bc]z+[bb]y &=[cl] \end{aligned}$$

The condition-equations should, therefore, be so arranged that as far as possible (and it cannot always be done in the first trial), the normal-equations will fall in form 1 rather than in form 2. A good rule is to begin with angle-equations, proceeding from triangle to triangle, until the points gone over are covered by a side-equation, and then introduce it. Repeat the process for the remaining sets of triangles.

As to the angle-equations themselves, it matters but little which triangles are taken to form them. It is better, however, to avoid triangles with very small angles, such as DEF in fig. Art. 51, and where angles so very small occur, to avoid triangles involving angles immediately contiguous to these small angles.

55. We may mention another thing which, though it causes no trouble in the solution, is a little perplexing, because unexpected. It is well known that the diagonal terms are always positive. Now in the formation of the normal equations from the correlate equations some of the terms that go to make up the diagonal terms may be negative. These terms are comparatively small and arise generally from angles in the neighborhood of 90°, especially if a little over 90°. It is a good plan, therefore, to avoid angles of 90°* in the formation of quadrilateral side-equations, the change in the log. sine being smallest at that point.

56. In the example chosen the selection of side and angle-equations may be made as follows, AB being taken as base:

* See C. S. Report, 1864.

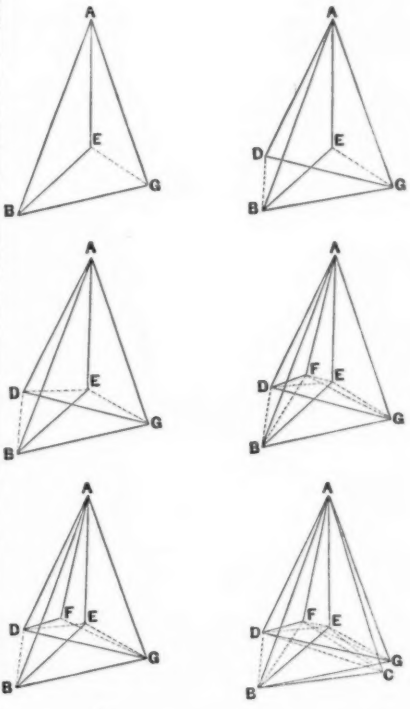
1. Triangle AGB
2. " ABE
3. " BEG

I. Quadrilateral AGBE

4. " DAG
5. " BDG

II. " AGBD

6. " DAE
(AEG=ABG−ABE−BEG)



III. Quadrilateral AGDE

7. Triangle DAF

IV. " AGDF

8. " BDF

V. " BDFG

9. " BFE

VI. " BFGE

10. " BAC

VII. " ACBE

VIII. " CBDE

Thus we have the ten angle-equations and the eight side-equations.

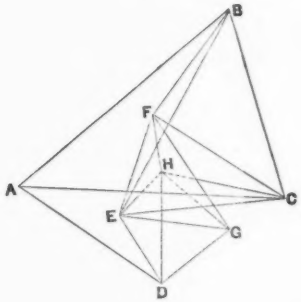
57. In the explanation of the formation of the side-equations we have assumed that the route from one side to the same side again in the triangulation-net furnishing a side-equation proceeds through a chain of triangles. This, however, is not always the case. A good illustration occurs in the triangulation of Lake Superior,* near Keweenaw Base, as shown in figure.

Here there are 18 lines and 8 points, requiring 5 side-equations. Proceeding from the line DG, and adding point to point, we have for the first four side-equations the quadrilaterals

HGDE, HGEF, HEFC, CEFB.

With these there is no difficulty.

The fifth side equation is furnished by the pentagon ABCED. We cannot compute directly any specified side of this pentagon from another side through triangles only. A little artifice, however, will enable us to do this. Suppose the line CD drawn. Call the angle $GDC=x$ and $ECD=y$. This new line CD gives an additional side-equation. We take the two pentagons ABCED, DEGFC. From the first pentagon ABCED



$$\frac{CE}{CD}\cdot\frac{CD}{CA}\cdot\frac{CA}{CB}\cdot\frac{CB}{CE}\cdot=1$$

or

$$\frac{\sin.\,EGD+x}{\sin.\,CDE}\cdot\frac{\sin.\,CAD}{\sin.\,ADG}\cdot\frac{\sin.\,CBA}{\sin.\,CAB}\cdot\frac{\sin.\,CEB}{\sin.\,CBE}\cdot=1$$

and from the second pentagon DEGFC

$$\frac{ED}{EG}\cdot\frac{EG}{EF}\cdot\frac{EF}{EC}\cdot\frac{EC}{ED}\cdot=1$$

or

$$\frac{\sin.\,EGD}{\sin.\,EDG}\cdot\frac{\sin.\,EFG}{\sin.\,EGF}\cdot\frac{\sin.\,(ECF+x)}{\sin.\,EFC}\cdot\frac{\sin.\,(EDG+x)}{\sin.\,y}=1 \qquad (84)$$

Now y can be expressed in terms of x from the triangle CDE. Thus

$$y+CED+EDG+x=180+\varepsilon \qquad (85)$$

Eliminating x from equations (84) and (85) the required side-equation results.

Now equation (84) is easily put in the form

$$\text{Cot.}\,(EDG+x)=\frac{1}{\sin.\,ADE}\cdot\frac{\sin.\,CAD}{\sin.\,CED}\cdot\frac{\sin.\,ABC}{\sin.\,BAC}\cdot\frac{\sin.\,BEC}{\sin.\,CBE}-\text{Cot.}\,ADE$$

	°	′	″	log. sin.
CAD =	24	28	40.92	9.6173614
ABC =	49	14	53.54	9.8794082
BEC =	64	40	04.26	9.9596005
				9.4563701
				9.3290804
				0.1272897
log. cot. ADE				0.1807050
				0.0534153

	°	′	″	log. sin.
ADE =	33	24	34.97	9.7408538
CED =	61	18	05.11	9.9430779
BAC =	53	16	36.98	9.9039225
CBE =	33	26	31.73	9.7412262
				9.3290804

Hence from Zech's Tables* it follows at once that

$$\text{log. cot.}\,(EDG+x)=9.2441624n$$

and

$$EDG+x=99^\circ\ 57''\ 04''.47$$

Hence since $\varepsilon=1''.08$ we easily find

$$3y=18^\circ\ 44'\ 51''.49.$$

All the angles of equation (85) being known the solution is carried through in the usual way.

Since a preliminary computation of the lengths of the sides for finding the spherical excess must be made, the angle x could be found roughly from the relation

$$\frac{EC}{\sin.\,(EDG+x)}=\frac{ED}{\sin.\,(180-EDG-x-CED-\varepsilon)}$$

* Report of Chief of Engineers, 1872.

* Tafeln der Addition and Subtraction Logarithmen. Von J. Zech.

where ε is the spherical excess of the triangle.

If chains of triangles intersect so as to enclose a polygon the preceding method may be applied in simple cases. It is, however, better treated by other methods. (See Zachariæ *Geodätische Hauptpunkte*, sec. 3; Clarke's *Geodesy*, pp. 255 *seq.*; *G. T. Survey of India*, vol. 2.)

58. *Closeness of computation necessary.*

As we know that exact values of the unknowns are never found, we may investigate how far it is allowable to cut short our computation. With a single unknown a method of computation is close enough which will give that unknown within $\frac{1}{10}$ of the mean square error μ expected. Taking this as a basis in a set of equations of the form

$$ax+ly+\ \ldots\ -l=v$$

and that is taking the allowable error of l to be $\frac{1}{10}\mu$, if we let σ be the average value of the greatest allowable error of ax, ly, . . then if m is the number of these terms we have

$$\frac{\mu^2}{100}>m\sigma^2$$

Now it will be a liberal estimate if we let $m=100$, so that the approximation is sufficient when

$$\sigma<\frac{\mu}{100}$$

The quantities ax, by, . . may be taken to be about 5μ numerically. Hence an approximation is sufficient where the error committed does not exceed $\frac{1}{500}$ of the value of the quantity. This can in general be carried out with a computation to 4 places of decimals, and if a log. table is used, a 5-place table is sufficient. In short systems 4-place tables will give results close enough.

[It does not lie within my province to explain the method of forming and solving normal-equations. This is done in the treatises already referred to. I may, however, remark that in the mechanical part of least squares, the day for the exclusive use of log. tables is past. The best aids are a table of squares, a table of reciprocals, a table of products, and a computing machine. No one who has worked with these tools will ever want to open a log. table for forming and solving normal-equations.]

THE LAKE SURVEY METHOD OF ADJUSTMENT.

59. The method of measuring horizontal angles employed on the Lake Survey is a modification of Bessel's method. It is as follows:

The instrument being in position, the signals around the horizon were sighted at in order of increasing graduation of the horizontal limb closing on the first signal and the microscopes read each time: then these pointings were repeated, but in the order of decreasing graduation, closing again on the first object. This gave one set.

The difference between consecutive pointings gave the included angle. The mean of an angle value in the direction of increasing graduation and an immediately following one in the opposite direction gave a combined result.

The telescope was then reversed, leaving the same pivot in the same wyes, and another set taken from which combined results are found.

The horizontal limb is now moved some aliquot part of 180°, and the preceding process repeated till microscope readings have been obtained at equidistant points entirely round the limb.

The number of sets before and after reversal should be the same. Probable errors are derived from combined results.

The resulting angles are assumed to be independent in the adjustment.

EXAMPLE. AT STATION D.

Circle left, positive.

Stat'n.	Micros. 1.	Mic. 2.	Mic. 3.	Mean.	Angle.
	° ′ ″	″	″	″	° ′ ″
A...	334 49 21.0	22.2	20.9	21.4	
B...	72 11 54.1	53.3	46.2	51.2	97 22 29.8
C...	250 25 51.8	54.1	55.1	53.7	178 14 02.5
	Circle left, negative.				
C...	51.3	52.9	54.0	52.7	
B...	54.0	53.0	46.0	51.0	01.7
A...	23.5	24.8	23.6	24.0	27.0
	Circle right, positive.				
A...	48.0	40.8	49.0	45.9	
B...	07.0	10.1	13.8	10.3	24.4
C...	14.9	16.0	10.1	13 7	3.4
	Circle right, negative.				
C...	14.3	14 5	9.8	12.9	
B...	7.2	9.7	13.7	10.2	2.7
A...	48.0	41.2	48.7	46.0	24.2

60. *Notation.*

The notation used on the Lake Survey was as follows: The stations were numbered from one end of the triangulation

to be adjusted to the other. The single angles in order of azimuth at each station were denoted by subscript numbers. Thus, at station 1 the angles, if all measured, would be denoted 1_1, 1_2, 1_3, 1_4, starting from the south point and going round by the west.

The local correction to 1_1 would be (1_1) and the general correction $[1_1]$. The measured sum-angle composed of the first two angles 1_1, 1_2 would be denoted by 1_{1+2}.

61. The method of computing the most probable corrections to the angles is almost precisely the same as that already explained in Arts. 19–21, and need not, therefore, be repeated. The example of Art. 34 will serve as an illustration.

Assuming the independence of the measured angles, this method of finding the corrections to them is perfectly rigorous.

EXAMPLE.

The Local Adjustment.

(*a*) At *North Base.*

The observation-equations

p	(1_1)	(1_2)	l
2	+1	+1	0
2		+1	0
14	−1	−1	−1.37

The normal-equations

(1_1)	(1_2)	
+16	+14	$=-19.18=[pal]_1$, suppose
+14	+16	$=-19.18=[pbl]_1$ "

Solving in general terms

$$(1_1)=+0.267\,[pal]_1-0.233[pbl]_1$$
$$(1_2)=-0,223\,[pal]_1+0.267[pbl]_1$$

Hence

$$(1_1)=-0.64$$
$$(1_2)=-0.64$$
$$(1_3)=+0.64+0.64-1.37=-0.09$$

and, most probable angles

°	′	″	
124	09	40.05	$+[1_1]$
113	39	04.43	$+[1_2]$
122	11	15.52	$-[1_1]-[1_2]$

(*b*) At *South Base.*

The observation-equations

p	2_1	2_2	l
23	+1		
6		+1	
7	+1	+1	−1.07

The normal-equations

(2_1)	(2_2)	
30	+ 7	$=-7.49$
7	+13	$=-7.49$

Solution in general terms

$$(2_1)=+0.038[pal]_2-0.021[pbl]_2$$
$$(2_2)=+0.021[pal]_2+0.088[pbl]_2$$

Most probable angles

°	′	″	
23	08	05.13	$+[2_1]$
47	31	19.91	$+[2_2]$
70	39	25.04	$+[2_1]+[2_2]$

The General Adjustment.

The Condition-Equations.

(*a*) Triangle. N. Base. S. Base. Oneota.

°	′	″	
122	11	15.52	$-[1_1]-[1_2]$
23	08	05.13	$+[2_1]$
34	40	39.66	$+[3_1]$
180	00	00.31	
$180+\varepsilon$ 180	00	00.06	

$$0=0.25-[1_1]-[1_2]+[2_1]+[3_1]$$

(*b*) Triangle. Lester, Oneota, S. Base.

°	′	″	
70	39	25.04	$+[2_1]+[2_2]$
78	27	06.06	$+[3_1]+[3_2]$
30	53	30.81	$+[4_1]$
180	00	01.91	
$180+\varepsilon=180$	00	00.37	

$$0=01.54+[2_1]+[2_2]+[3_1]+[3_2]+[4_1].$$

(*c*) Quadrilateral. N. Base, S. Base, Lester, Oneota.

$$\frac{\sin.\,LNS}{\sin.\,LNO}\cdot\frac{\sin.\,LOS}{\sin.\,NSL}\cdot\frac{\sin.\,LON}{\sin.\,LOS}=1$$

° ′ ″

113 39 04.43 + [1$_3$]
70 39 25.03 + [2$_1$] + [2$_2$]
43 46 26.40 + [3$_3$]

log. sin.	diff. 1.″
9.9618975.6 −	9.22
9.9747660.1 +	7.39
9.8399903.4 +	21.98

539.1
509.4

29.7

° ′ ″

124 09 40.05 + [1$_1$]
47 31 19.90 + [2$_2$]
78 27 06.06 + [3$_1$] + [3$_3$]

log. sin.	diff. 1.″
9.9177479.3 −	14.29
9.8677849.8 +	19.28
9.9911180.3 +	4.30

509.4

A glance at the log. differences for 1″ shows that by expressing them in units of the sixth place of decimals their average value is unity. We have then the side-equation

$$1.43[1_1] - 0.92[1_3] + 0.74[2_1] - 1.19[2_2] - 0.43[3_1] + 1.77[3_3] + 2.97 = 0$$

The weight equations.

$$[1_1] = -0.233[pal]_1 + 0.267[pbl]_1$$
$$[1_3] = +0.267[pal]_1 - 233[pbl]_1$$
$$[2_1] = +0.038[pal]_2 - 0.021[pbl]_2$$
$$[2_2] = -0.021[pal]_2 + 0.088[pbl]_2$$
$$[3_1] = +1.000[pal]_3$$
$$[3_3] = +0.032[pal]_4$$
$$[4_1] = +0.125[pal]_5$$

The correlate-equations.

	I.	II.	III.
$[pal]_1 =$	−1	0	+1.43
$[pbl]_1 =$	−1	0	−0.92
$[pal]_2 =$	+1	+1	+0·74
$[pbl]_2 =$	0	+1	−1.19
$[pal]_3 =$	0	+1	+1.77
$[pal]_4 =$	0	+1	−0.43
$[pal]_5 =$	0	+1	0.

The corrections in terms of the correlates

	I.	II.	III.
$[1_1] =$	−0.034	0	+0.596
$[1_3] =$	−0.034	0	−0.569
$[2_1] =$	+0.038	+0.017	+0.052
$[2_2] =$	−0.021	+ 067	−0.121
$[3_1] =$	0.	+1.	−0.014
$[3_3] =$	+0.032	+0.032	+1.770
$[4_1] =$	0.	+0.125	0.

The normal-equations

I.	II.	III.		
+0.138	+0.049	+0.021	=	−0.250
	+1.241	+1.687	=	−1.540
		+4.706	=	−2.970

The solution of these equations gives

I. = −1.522
II. = −0.647
III. = −0.392

Substitute for I., II., III., in the correlate-equations we have the general corrections. Adding the station corrections and general corrections together, the total corrections to the measured angles are:

	(v)	[v]	v	p	pvv	Final Angles.
	″	″	″			° ′ ″
1_1	= −0.64	−0.18 =	−0.82	2	1.34	124 09 39.87
1_2	= −0.64	+0.28 =	−0.36	2	.26	113 39 4.71
1_3	= −0.09	−0.10 =	−0.19	14	.50	122 11 15.42
2_1	= −0.13	−0. 9 =	−0.22	28	1.10	23 08 5.04
2_2	= +0.04	−0.50 =	−0.46	6	1.37	47 31 19.95
2_1+_2	= +0.44	−0.05 =	+0.39	7	1.06	70 39 24.99
3_1	=	−0.06 =	−0.06	31	0.12	34 40 39.60
3_2	=	−1.33 =	−1.33	1	1.77	43 46 25.07
4_1	=	−0.08 =	−0.08	8	.05	30 53 30.73
					7.47	

62. *The Precision determination.*

In the precision determination, an approximate method was employed.

For the p. e. of an angle of weight unity, we have the usual formula,

$$r = .6745\sqrt{\frac{[pvv]}{n_c}}$$

when n_c is the number of conditions, local and general.

For the p. e. of an angle in the adjusted figure, an average value is found from formula (25); thus, if

r' = p. e. of an adjusted angle which before adjustment had weight unity,
n_o = whole number of angles observed.

Then

$$r' = r\sqrt{\frac{n_o - n_c}{n_o}} \quad . \quad . \quad . \quad (86).$$

For the p. e. of a side, a single chain of the best shaped triangles, and as direct as possible between the base and the side is selected; all tie lines being rejected.

Suppose in a chain, b is the base and a_n the side, whose p. e. is to be found, then

$$a_n = b.\frac{\sin.A_1 \sin.A_2 \ldots\ldots}{\sin.B_1.\sin.B_2 \ldots\ldots}$$

or taking logs. and reducing to the linear form, as in

$$d\log.a_n = d\log.b + \frac{d.\log.\sin.A_1}{d\,A_1}.(A_1) + \ldots$$

$$-\frac{d\log.\sin.B_1}{d\,B_1}(B_1) - \ldots.$$

Now assume that

(1) b is exact.
(2) A_1, B_1, ... are all independently observed,
(3) these observed angles have a common p. e. r'.

Then

$$(\text{p.e.})^2 \log.a_n = [\delta^2_A + \delta^2_B]r'^2.$$

When δ_A, δ_B . . . . denote the change of log. sin. A_1, log. sin. B_1, ... for the angles A_1, B_1, ... in a table of log. sines.

Example.—To find the p. e. of OL as derived from NS. (See Fig. Art. 34).

The chain of triangles is ONS, OLS.
Number of angles = 9.
From the adjustment, we find

$$[pvv.] = 7.5.$$

No. of conditions, local and general, = 5

$\therefore$ p. e. of an angle of weight 1

$$= 0.6745\sqrt{\frac{7.48}{5}}$$

$$= 0''.82$$

p. e. of an angle of average weight

$$= \sqrt{\frac{9-5}{9}}.r = \frac{2}{3}r$$

Station.	Angle.	Diff. 1″ in units of 7th dec. place.	Diff'ce².
N. Base.	122° 11′ 15″	−13.2	174.2
S. Base.	23 08 05	49.3	2430.5
Oneota..	34 40 40	30.4	924.2
S. Base .	70 39 25	7.4	54.8
Oneota..	78 27 05	4.3	18.5
Lester..	30 53 30	35.2	1239.0
		$[a^2+\beta^2]$	=4841.2

$\therefore$ p. e. log. a_n = 38.27 in units of the seventh place of decimals.

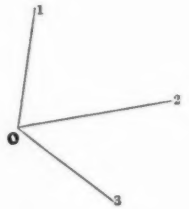
The Method of Directions.

63. I shall take up the usual case in which the directions are observed in "arcs," as explained in Art. 32, and give Bessel's method of reduction as modified and extended by Hansen and Andrae, whose changes have been generally adopted in the more modern European surveys.

The adjustment is divided into two parts, the station adjustment and the general adjustment.

The Station Adjustment.

64. Let O be the station occupied and 1, 2, 3. . . the stations sighted at in order of azimuth.



Let some one direction be selected as the zero direction, and let A_1, B_1, ... denote the most probable values of the angles which the directions of the different signals make with this direction.

In the first arc let X^1 denote the most probable value of the angle between the zero of the limb of the instrument and the direction of the signal taken as the zero direction, then if M_1', M_1'' . . . denote the readings of the limb for different signals and V_1', V_1'' . . . the most probable corrections to these readings, we have the observation equations

$$X'-O-M_1'=V_1'$$
$$X'-A-M_1''=V_1''$$
$$X'-B-M_1'''=V_1'''.$$
$$\ldots\ldots\ldots\ldots$$

The zero of the limb being changed in the next arc, we have in like manner

$$X''-O-M_2'=V_2'$$
$$X''-A--M_2''-V_2''$$
$$X''-B-M_2'''=V_2'''$$
$$\ldots\ldots\ldots\ldots$$

and so on.

Let $X'=X_0'+x' \qquad A=A_0+(A)$
$X''=X_0''+x'' \qquad B=B_0+(B)$
. . . .

when $X_0', X_0'' \ldots A_0, B_0 \ldots$ are approximate values of $X', X'' \ldots A, B, \ldots$ and $x', x'', \ldots (A), (B) \ldots$ denote their most probable corrections.

Also for convenience in writing, put

$m_1'=M_1'-X_0' \qquad m_2'=M_2'-X_0''$
$m_1''=M_1''-X_0'-A_0 \qquad m_2''M_2''-X_0''-A_0$
. . . .

The observation equations may now be written

$$\begin{aligned} x'-m_1' &= v_1' & x''-m_1' &= v_2' \\ x'+(A)-m_1''&=v_1'' & x''+(A)-m_2''&=v_2'' \\ x'+(B)-m_1'''&=v_1''' & x''+(B)-m_2'''&=v_2''' \\ \ldots\ldots\ldots & & \ldots\ldots\ldots & \end{aligned} \qquad (87).$$

If now $p_1', p_1'' \ldots : p_2' p_2'' \ldots$ are the weights of the measured directions of the several series of arcs, the normal-equations follow at once from the above derived observation-equations.

They are

$$\begin{aligned} [p_1]x' \quad &+p_1''(A)+p_1'''(B)+\ldots=[p_1m_1] \\ [p_2]x'' \quad &+p_2''(A)+p_2'''(B)+\ldots=[p_2m_2] \\ &\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad (88).$$

$$\begin{aligned} p_1''x'+p_2''x''+\ldots+[p'']A) &=[p''m''] \\ p_1'''x'+p_2'''x''+\ldots+[p''']B) &=[p'''m'''] \\ &\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

The quantities $x', x'' \ldots$ being merely auxiliary quantities, we eliminate them by substituting their values as found from the first group of normal-equations in the second group.

We have then

$$\left\{[p'']-\frac{p_1''}{[p_1]}p_1''-\frac{p_2''}{[p_2]}p_2''-\ldots\right\}(A)$$
$$+\left\{-\frac{p_1''}{[p_1]}p_1''-\frac{p_2''}{[p_2]}p_2''-\ldots\right\}(B)+\ldots$$
$$=[p''m'']-\frac{p_1''}{[p_1]}-pm-\frac{p_2''}{[p_2]}[p_2m_2]\ldots.$$
$$+\left\{[p''']-\frac{p_1'''}{[p_1]}p_1'''-\frac{p_2'''}{p_2}p_2'''-\ldots\right\}(B)$$
$$+\ldots=[p'''m''']-\frac{p_1'''}{[p_1]}[p_1m_1]$$
$$-\frac{p_2'''}{[p_2]}[p_2m_2]\ldots.$$
$$\ldots\ldots\ldots\ldots\ldots$$

which may for convenience be written

$$\begin{aligned} [aa](A)+[ab](B)+\ldots.&=[al] \\ [ab](A)+[bb](B)+\ldots.&=[bl] \\ &\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad (89).$$

where $[aa], [ab] \ldots$ are to be looked on as mere symbols.

65. If the original observations are arranged in groups for adjustment, as is usually done, then

$$p_1'=p_1''=\ldots\ldots\ldots$$
$$p_2'=p_2''=\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$

and if we put

$$[p_1]=\nu_1 p_1, [p_2]=\nu_2 p_2 \ldots\ldots\ldots$$

$\nu_1, \nu_2 \ldots$ being the numbers of observations in the respective groups, we have for the coefficients of the normal-equations

$$[aa]=p''-\frac{p_1''}{p_1}\cdot\frac{p_1''}{\nu_1}-\frac{p_2''}{p_2}\cdot\frac{p_2''}{\nu_2}-\ldots\ldots$$

$$[ab]=-\frac{p_1''}{p_1}\cdot\frac{p_1'''}{\nu_1}-\frac{p_2''}{p_2}\cdot\frac{p_2'''}{\nu_2}-\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots \qquad (90).$$

$$[al]=p''m''-\frac{p_1''}{p_1}\cdot\frac{[p_1m_1]}{\nu_1}$$
$$-\frac{p_2''}{p_2}\cdot\frac{[p_2m_2]}{\nu_2}-\ldots.$$

$$[bb]=p'''-\frac{p_1'''}{p_1}\cdot\frac{p_1'''}{\nu_1}-\frac{p_2'''}{p_2}\cdot\frac{p_2'''}{\nu_2}-\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$

$$[bl]=[p'''m''']-\frac{p_1'''}{p_1}\cdot\frac{[p_1m_1]}{\nu_1}$$
$$-\frac{p_2'''}{p_2}\cdot\frac{[p_2m_2]}{\nu_2}-\ldots.$$

66. *Checks of the Normal-Equations.* 1.—The sum

$$\begin{aligned} &[aa]+[ab]+[ac]+\ldots\ldots\ldots \\ &+[bb]+[bc]+\ldots\ldots\ldots \\ &+[cc]+\ldots\ldots\ldots \\ &+\ldots\ldots\ldots \end{aligned}$$

is equal to half the number of observations, less half the number of arcs. (91.)

For substitute for $[aa], [ab] \ldots$ their values from (89) we find the above sum equal to

$$[p'']+[p''']+\ldots-\frac{p_1'''^2+p_1''''^2+p_1''p_1'''+\ldots}{[p_1]}$$

$$-\frac{p_2'''^2+p_2''''^2+p_2''p_2'''+\ldots}{[p_2]}-\ldots.$$

$$=\frac{1}{2}\Big\{[p']+[p'']+[p''']+\ldots\Big\}$$

$$+\frac{-p_1'+p_1''+p_1'''+\ldots}{2}$$

$$-\frac{p_1'''^2+p_1''''^2+p_1''p_1'''+\ldots}{[p_1]}$$

$$+\frac{-p_2''+p_2''+p_2'''+-}{2}$$

$$-\frac{p_2'''^2+p_2''''^2+p_2''p_2'''+\ldots}{[p_2]}$$

$$+\ldots\ldots\ldots\ldots\ldots\ldots$$

$$=\frac{1}{2}\Big\{[p']+[p'']+[p''']+\ldots\Big\}$$

$$-\frac{[p_1^2]}{2[p_1]}-\frac{[p_2^2]}{2[p_2]}-\ldots\ldots\ldots$$

Now $p_1'=p_2'=\ldots.=1$ and $\therefore$

$[p']+[p'']+[p''']+\ldots=$ the number of observations.

$$\frac{[p_1^2]}{[p_1]}+\frac{[p_2^2]}{[p_2]}+\frac{[p_3^2]}{[p_3]}+\ldots=\text{the number of arcs.}$$

2.—The sum

$$[al]+[bl]+[cl]+\ldots.=[wl] \quad (92).$$

where $[wl]$ is formed in the same way as $[al]$, $[bl]$, ... For

$$[al]+[bl]+\ldots=[p''m'']+[p'''m''']+\ldots$$

$$+[p_1m_1]\left(\frac{p_1'}{[p_1]}-1\right)+[p_2m_2]\left(\frac{p_2'}{[p_2]}-1\right)+\ldots$$

$$=\frac{p_1'}{[p_1]}[p_1m_1]+\frac{p_2'}{[p_2]}+[p_2m_2]+\ldots.$$

Since $[p''m'']+[p'''m''']+\ldots.$

$$-[p_1m_1]-[p_2m_2]-\ldots=[p'm']=0$$

which proves the proposition.

After having found the quantities $m', m''\ldots.$, by taking the differences between the approximate values of the angles and the several measured values, it is convenient to arrange the formation of the normal-equations according to the following scheme:

No. of group.	$\frac{p}{\nu}$	p	$p''m''$	$p'''m'''$	..	$[pm]$	$\frac{[pm]}{\nu}$
1	$\frac{p_1}{\nu_1}$	p_1	$p_1''m_1''$	$p_1'''m_1'''$	..	$[p_1m_1]$	$\frac{[p_1m_1]}{\nu_1}$
2	$\frac{p_2}{\nu_2}$	p_2	$p_2''m_2''$	$p_2'''m_2'''$	..	$[p_2m_2]$	$\frac{[p_2m_2]}{\nu_2}$
3	$\frac{p_3}{\nu_3}$	p_3	$p_3''m_3''$	$p_3'''m_3'''$	..	$[p_3m_3]$	$\frac{[p_3m_3]}{\nu_3}$
..	..	..	....	....	..	..	..
		$[p]$	$[p''m'']$	$[p'''m''']$	..	$[pm]$	

The coefficients of the normal-equations may now be written down at sight.

67. *To find the m. s. e. of a single observed direction.*

We have

$$\mu^2=\frac{[pvv]}{\text{No. of obs. quan's}-\text{No. of indep't unknowns}}$$

Now, from equations (81), it is evident that the number of observation-equations is equal to the number of directions n_d, and then umber of independent unknowns is the number of arcs n_a + the number of stations or signals n_s less one. Hence

$$\mu^2=\frac{n_d-(n_a+n_s-1)}{[pvv]} \quad (93).$$

To compute the value of $[pvv]$.

From equations (88) and (89) we have the reduced normal-equations,

$$[p_1]x'+\quad +p_1''(A)+p_1'''(B)+\ldots=[p_1m_1]$$
$$[p_2]x''\quad +p_2''(A)+p_2'''(B)+\ldots=[p_2m_2]$$
$$\ldots\ldots\ldots\ldots$$
$$[aa](A)+[ab](B)+\ldots\ldots=[al]$$
$$[bb1](B)+\ldots\ldots=[bl.1]$$
$$\ldots\ldots\ldots\ldots$$

Hence, as in Art. 13,

$$[pvv]=[pmm]-\frac{[p_1m_1]^2}{[p_1]}-\frac{[p_2m_2]^2}{[p_2]}-\ldots.$$
$$-\frac{[al]^2}{[aa]}-\frac{[bl.1]^2}{[bb.1]}-\ldots. \quad (94).$$

$$=[pmm]-\frac{[p_1m_1]^2}{[p_1]}-\frac{[p_2m_2]^2}{[p_2]}-\ldots.$$
$$-[al](A)-[bl](A)-\ldots \quad (95).$$

The quantity $[pmm]$ should always, if possible, be found from the original observations. It will generally be quite different if found from the means forming

the different groups of arcs taken as single observations of a certain weight.

When found from the original observations,

$$p_1'=p_1''=\ldots\ldots=1$$
$$p_2'=p_2''=\ldots\ldots=1$$
$$\ldots\ldots\ldots\ldots$$

and then

$$[vv]=[mm]-\frac{[m_1]^2}{\nu_1}-\frac{[m_2]^2}{\nu_2}-\ldots.$$
$$[al](\mathbf{A})-[bl](\mathbf{B})\ldots.(96).$$

Check formula.—It is evident from the above expression that the computation of the value of $[vv]$ falls into two parts, corresponding to the two sets into which the normal-equations are divided.

A check of the second part is afforded by the equality of the values resulting from

$$\frac{[al]^2}{[aa]}+\frac{[bl1]^2}{[bb1]}+\ldots\ldots\ldots$$
$$[al](\mathrm{A})+[bl](\mathrm{B})+\ldots..$$

The first part which may be denoted by $[vv]_e$ may also be computed in two ways. For

$$[vv]_e=[mm]-\frac{[m_1]^2}{\nu_1}-\frac{[m_2]^2}{\nu_2}-\ldots.(97).$$
$$=m_1'm_1'+m_1''m_1''+\ldots.$$
$$+m_2'm_2'+m_2''m_2''+\ldots.$$
$$-2\frac{[m_1]}{\nu_1}(m_1'+m_1''+..)$$
$$-2\frac{[m_2]}{\nu_2}(m_2'+m_2''+\ldots..)$$
$$+\frac{[m_1]^2}{\nu_1^2}+\frac{[m_1]^2}{\nu_1^2}+\ldots\ldots\ldots \text{ to } \nu_1 \text{ terms.}$$
$$+\frac{[m_2]^2}{\nu_2^2}+\frac{[m_2]^2}{\nu_2^2}+\ldots\ldots\ldots \text{ to } \nu_2 \text{ terms.}$$
$$+\ldots\ldots\ldots\ldots\ldots\ldots$$
$$=\left(m_1'-\frac{[m_1]}{\nu_1}\right)^2+\left(m_1''-\frac{[m_1]}{\nu_1}\right)^2+..$$
$$+\left(m_2'-\frac{[m_2]}{\nu_2}\right)^2+\left(m_2''-\frac{[m_2]}{\nu_2}\right)^2+..$$
$$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (98).$$

The Coast Survey Method of Adjustment.

68. The method of observation of directions in the primary triangulation is essentially that of Bessel. "The number of observations required for the determination of any one angle must depend upon the desired closeness of the result and the character of the instrument." It is left entirely to the judgment of the observer.

"The direction instrument requires that it should be turned on its stand or changed in position in order that the direction of any one line, and consequently of all, should fall upon different parts of the circle as the only security against errors of graduation. The number of positions varies from five to twenty-one of nearly equal arcs: and in each position the circuit of the horizon is made giving the direction of each line by two observations, one in the direct and the other in the reversed position of the telescope. These circuits or series are repeated in each position until two to five values of each direction are obtained. Each angle is therefore determined by from 35 to 63 measurements in the direct, and a like number in the reversed position of the telescope."*

69. The adjustment† is broken into two parts as Bessel's method requires. The total correction to any observed angle M, is therefore made up of two corrections x and y, the one resulting from the adjustment of the directions at the station occupied, the other from the geometrical conditions existing in the triangulation figure. The method of least squares requires

$$[(x+y)^2]=\text{a minimum}$$

which is assumed to be identical with making $[x^2]$ and $[y^2]$ respectively a minimum.

The measured angles at a station are first adjusted, and with these adjusted values $\mathrm{M}+x$, we enter the angle and side-equations which determine the further corrections y, no subsequent notice of the station equations being taken.

This, though not strictly correct, may be considered practically good enough. In good work there will be no marked disarrangement of the station conditions arising from the net adjustment.

The Local Adjustment.

70. The method of deriving the station normal-equations with the checks used is substantially that already given in Arts. 64–65. The quantities m_1' m_1''.... are called "diminished measures."

* Methods, Discussions, and Results, Field work of the Triangulation, 1877.
† Report 1854, App. 33: Report 1864, App. 14.

The tabular arrangement for carrying out the numerical work is slightly different, and will now be given for the following

EXAMPLE—AT CLARK MT.

Spear.	Humpback.	Fork.
	° ′ ″	° ′ ″
00 00 00.000	24 09 35.70	78 26 08.55
.00	39.40	09.60
.00	36.23	09.33
.00	33.55	10.45
00 00 00.00		78 26 10.20
.00		11.03
00.00	24 09 36.10	
.00	37.40	
.00	36.53	
.00	38.15	
.00	39.00	
	00 00 00.00	54 16 31.85
		31.94
		32.03
		36.16

Assume the most probable value of the angles.

	° ′ ″
Spear.....	00 00 00.00
Humpback....	24 09 36.90 + (A)
Fork.....	78 26 09.90 + (B)

First form the abstract of diminished measures, by taking the difference between each measured value, and the most probable values of the angles. This gives m_1', m_1''.... To facilitate this work the complements of the seconds are formed for each initial direction, and thus the subtractions are changed to additions. We have then the auxiliary table:

Station.	Assumed angles.	Complements.	Quantities for addition.
Spear.....	00 00 00.00	00.00	
Humpback..	24 09 36.90	23.10	00.00
Fork.....	78 26 09.90	50.10	27.00

In tabulating the abstract of diminished measures, the series of an equal number of directions are placed together beginning with the greatest number. The sums and means are next found as indicated below, and a check is afforded by finding $[pm]$ from the sums in vertical and horizontal rows. (See Table, following page.)

Next form the coefficients of the normal-equations. They are:

$$[aa] = 13 - \tfrac{4}{3} - \tfrac{2}{3} = 7\tfrac{1}{3}$$
$$[bb] = 10 - \tfrac{4}{3} - \tfrac{2}{3} = 5\tfrac{2}{3}$$
$$[ab] = -\tfrac{4}{3} - \tfrac{4}{3} = -3\tfrac{1}{3}$$
$$[al] = -0.04 + 1.4633 + 0.025 - 1.340 = +0.1083$$
$$[bl] = -0.29 + 1.4633 - 0.715 - 0.025 = +0.4833$$
$$[\omega l] = -1.4633 + 0.715 + 1.34 = +0.5917$$

Check (1) $[aa] + [ab] + [bb] = 9\tfrac{1}{2} = 17 - 7\tfrac{1}{2}$
$= \tfrac{1}{2}$ (No. of obs. − No. of ares.)

(2) $[al] + [bl] = 0.5916 = [\omega l]$ as it should.

Hence the normal-equations are:

(A)	(B)	
$+7\tfrac{1}{3}$	$-3\tfrac{1}{3}$	$= +0.1083$
$-3\tfrac{1}{3}$	$+5\tfrac{2}{3}$	$= +0.4833$

Solving we have

(B) = 0.130 (A) = 0.075

and therefore the adjusted values of the angles are

	° ′ ″
Spear.....	00 00 00.000
Humpback.....	24 09 36.975
Fork.....	78 26 10.030

So far the method of computation has been rigorous. In what follows to prevent misunderstanding I have frequently quoted directly from the Reports.

71.—We proceed to determine the probable error of each direction.

Now "if s equal the number of observations of any one direction the formula

$$r_1^2 = \frac{.455 \Sigma v^2}{s(s-1)}$$

gives an approximation to the probable error of a direction; to give, however, proper weights to the results since the v's of the more full series are more correct than of series with less directions, it is preferable to substitute for $s-1$ the diagonal coefficient of the weight equations (omitting the remaining combinations) this gives for the

p.e. of the first direction $r_1 = \sqrt{\frac{.455 \Sigma v^2}{s[\omega\omega]}}$

" " second " $r_1 = \sqrt{\frac{.455 \Sigma v^2}{s[aa]}}$

" " third " $r_1 = \sqrt{\frac{.455 \Sigma v^2}{s[bb]}}$

etc.; (99).

ABSTRACT OF DIMINISHED MEASURES.

Spear.	Humpback.	Fork.		
00.00	58.80	58.65	$\nu_1=3$	
.00	2.50	59.70		
.00	59.33	59.42		
.00	56.65	00.55		
	$[p_1''m_1'']=-2.72$	$[p_1'''m_1''']=-1.67$	$[p_1m_1]=-4.39$	$\frac{[p_1m_1]}{\nu_1}=-1.4633$
0.00		.30	$\nu_2=2$	
.00		1.13		
		$[p_2'''m_2''']=+1.43$	$[p_2m_2]=+1.43$	$\frac{[p_2m_2]}{\nu_2}=+0.7150$
0.00	59.20			
.00	.50			
.00	59.63			
.00	1.25			
.00	2.10			
	$p_3''m_3''=+2.68$		$[p_3m_3]=+2.68$	$\frac{[p_3m_3]}{\nu_3}=-1.3400$
	00.00	58.85		
	.00	58.94		
	.00	59.03		
	.00	3.13		
		$p_4'''m_4'''=-0.05$	$[p_4m_4]=-0\ 05$	$\frac{[p_4m_4]}{\nu_4}=-0.0250$
Sums	$[p''m'']-0.04$	$[p'''m''']=-0.29$	$[pm]=-0.33$	Check sum.
$[p']=11$	$[p'']=13$	$[p''']=10$	$[p]=34$	
ω	a	b		

we also have weight $p=\frac{1}{r_1^2}$."

Here $[aa],[bb]\ldots$ are as in (68), and

$$[\omega\omega]=[p']-\frac{p_1'^2}{[p_1]}-\frac{p_2'^2}{[p_2]}\cdot\ldots$$

$$=[p']-\frac{p_1'}{p_1}\cdot\frac{p_1'}{p_1}-\frac{p_2'}{p_2}\cdot\frac{p_2'}{v_2}\ldots(100).$$

Check of the computation of $[\omega\omega]$

$$[\omega\omega]=[p']-\frac{p_1'^2}{[p_1]}-\frac{p_2'^2}{[p_2]}\cdot\ldots\ldots$$

$$[aa]=[p'']-\frac{p_1''^2}{[p_1]}-\frac{p_2''^2}{[p_2]}\cdot\ldots\ldots$$

$$[bb]=[p''']-\frac{p_1'''^2}{[p_1]}-\frac{p_2'''^2}{[p_2]}\cdot\ldots\ldots$$

$$\therefore \text{Sum}=[p']+[p'']+\ldots\frac{[p_1^2]}{[p_1]}-\frac{[p_2^2]}{[p_2]}.$$

= the number of observations, − the number of arcs. (101).

In our example

$$[\omega\omega]=11-\tfrac{4}{3}-\tfrac{8}{2}-\tfrac{5}{2}=6\tfrac{1}{6}$$

Check

$$[\omega\omega]+[aa]+[bb]=6\tfrac{1}{6}+7\tfrac{1}{6}+5\tfrac{2}{3}$$
$$=19$$
$$=34-15.$$

It is necessary next to find $v_1^2, v_2^2, \ldots$ We have

$$v_1'=x'-m_1' \qquad v_2'=x''-m_2''$$
$$v_1''=x'+A-m_1'' \qquad v_2''=x''+A-m_2'''$$
$$v_1'''=x'+B-m_1''' \qquad v_2'''=x''+B-m_2''$$
$$\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots$$

which may be written

$$v_1'=x'-d_1' \qquad v_2'=x''-d_2'$$
$$v_1''=x'-d_1'' \qquad v_2''=x''-d_2''$$
$$v_1'''=x'+d_1''' \qquad v_2'''=x''-d_2'''$$
$$\ldots\ldots\ldots \qquad \ldots\ldots\ldots$$

Now from the first set of normal-equations

$$p_1x'=p_1'(m_1'-o)+p_1''(m_1''-(A))+p_1'''(m_1'''-(B))+\ldots$$
$$=p_1'.d_1'+p_1''.d_1''+\ldots$$
$$p_2x''=p_2'.d_1'+p_2''d_2''+\ldots$$
$$\ldots\ldots\ldots\ldots$$

But $p_1'=p_1''\ldots=p_1$

and $[p_1]=p_1.\nu_1$

$$\therefore \quad x' = \frac{[d_1]}{\nu_1} \qquad x'' = \frac{[d_2]}{\nu_2}.$$

$$\text{and} \quad v_1' = \frac{[d_1]}{\nu_1} - d_1' \qquad v_2' = \frac{[d_2]}{\nu_2} - d_2''$$

$$v_1'' = \frac{[d_1]}{\nu_1} - d_1'' \qquad v_2'' = \frac{[d_2]}{\nu_2} - d_2''$$

. . . .

The quantities d_1, d_2 . . are called *remaining differences* and v_1', v_2 . . *remaining errors.* The tables used in computing these quantities are called the *abstract of remaining differences* and the *abstract of remaining errors*, respectively.

We have ∴ to pass through two steps in finding the remaining errors:

(1.) The abstract of remaining differences d_1, d_2 . . referred to the initial direction of each series is found by applying the corrections (A), (B) . . . with their signs changed to the respective quantities in the abstract of diminished measures. For another initial direction than the first, the proper differences must be applied. The computation is carried out to the nearest tenth of a second only.

(2.) The abstract of remaining errors v_1, v_2 . . . is formed by comparing each difference d_1, d_2 . . . of the abstract of remaining differences with the mean of its arc. It contains the quantities of the form

$$\frac{[d]}{\nu} - d$$

that is the remaining errors.

ABSTRACT OF REMAINING DIFFERENCES.

Spear.	Humpback.	Fork.	$\frac{d}{v}$
	′	″	
00 0	58.7	58.5	59.1
0	2.4	59.6	.7
0	59.3	59.3	59.5
0	56.6	00.4	59.0
0.0		0.2	0.1
0		1.0	0.5
0.0	59.1		59.6
.0	.4		2
.0	59.6		59.8
.0	1.2		.6
.0	2.0		1.0
	0.0	58.8	59.4
	.0	58 9	59.4
	.0	59.0	59.5
	.0	3.1	1.6

ABSTRACT OF REMAINING ERRORS.

Spear.		Humpback.		Fork.	
v^2	v^2	v^2	v^2	v	v^2
.9	.8	.4	.2	.6	.4
.7	.5	1.7	2.9	1.1	1.2
.5	.2	.2	0	.2	.0
1.0	1.0	2.4	5.8	1.4	2.0
	2.5		8.9		3.6
.1	.0			.1	0.
.5	.2			.5	.2
	.2				.2
.4	.2	.5	.2		
.2	.0	.2	.0		
.2	.0	.2	.0		
.6	.4	.6	.4		
1.0	1.0	1.0	1.0		
	1.6		1.6		
		.6	.4	.6	.4
		.6	.4	.5	.2
		.5	.2	.5	.2
		1.6	2.6	1.5	2 2
			.36		3.0
	4.3		14.1		6.8

	Spear.	Humpback.	Fork.
s	11.	13.	10.
$s[\ ]$	67.8	93.2	56.7
$.455\ v^2$	1.96	6.42	3.09
r^2	0.029	.069	.054
p	34.5	14.5	18 5
r	0.17	0.26	0.23

72. We have thus at △ Clark Mt. found the corrections x to each direction and also the probable error r, of each direction. In the same way at the remaining stations, Spear, Humpback, Fork, of the quadrilateral CSHF which we intend to consider, the corrections have been found from the angles employed. The corrected values for the four stations with the value of r for each direction are as follows:

		°	′	″	″
At Clark Mt.	(1) Spear....	00	00	00.000	± 0.17
	Humpback...	24	09	36.975	± 0.26
	Fork.....	78	26	10.030	± 0.23
At Spear	(2) Humpback...	00	00	00.000	± 0.23
	Fork.....	32	08	11.793	± 0.21
	Clark Mt.....	54	06	29.197	± 0.29

	°	′	″	″
At Humpback (3)Clark Mt.	00	00	00.000	±0.19
Spear	101	44	03.123	±0.33
Fork	332	58	11.157	±0.27
At Fork (4) Clark Mt	00	00	00.000	±0.14
Spear	79	35	42.479	±0.33
Humpback ..	98	41	43.926	±0.20

73. It remains now in adjusting the quadrilateral to find the corrections y arising from the side and angle-equations.

The quadrilateral includes four triangles. If S denote the sum of the three angles of a triangle, then $S-180-\varepsilon$ denotes the error of closure of a triangle. The spherical excess and closing error of each triangle are given in the following table. Advantage may be taken of $S-180-\varepsilon$ in forming the angle-equations.

Triangle.	Excess. ″	$S-180-\varepsilon$. ″	$(S-180-\varepsilon)^2$
241	10.773	0.860	0.7396
341	7.386	1.562	2.4398
234	6.402	1 196	1.4304
231	9.789	0.494	0.2440
			Sum 4.8538

Hence
probable error in a triangle=

$$.6745\sqrt{\frac{4.8538}{4}}$$

$$=\pm 0''.743$$

$$\text{probable error in an angle}=\frac{\pm 0.743}{\sqrt{3}}$$

$$=\pm 0''.429$$

$$\text{probable error in a direction}=\frac{\pm 0.429}{\sqrt{2}}$$

$$=\pm 0''.303$$

"These quantities include the errors of observation as well as that arising from the formation of triangles." We therefore write for the probable error of a direction

$$r_{1+\Delta}=\pm 0.''303.$$

"The combination of the angles to triangles introduces a new kind of error which must be taken into account: its general effect is to equalize the weights. This triangle error may be supposed to be principally due to lateral refraction; minor causes may be recognized in the imperfect centering of the theodolite over the same vertical observed upon from the other two stations, and in want of parallelism in the verticals at two stations of unequal elevation, and perhaps in other circumstances. If we separate the observing error r_1 from the error found above, $r_1+\Delta$, we have the triangle error r_Δ. To do this, the probable error of the 24 directions forming the quadrilateral are tabulated and the mean taken. We have

Triangle.	Direction.	Prob. error. ″	Triangle.	Direction.	Prob. error. ″
124	12	±0.17	234	23	±0.23
	14	.23		24	.21
	21	.29		32	.33
	24	.21		34	.27
	41	.14		42	.33
	42	.33		43	.20
123	12	±0.17	134	31	±0.19
	13	.26		34	.27
	21	.29		41	.14
	23	.23		43	.20
	31	.19		13	.26
	32	.34		14	.23

The average of these values gives

$$r_1=\pm 0.238.$$

But

$$r_{1+\Delta}=\pm 0.303.$$

Hence,

$$r_\Delta=\sqrt{0.303^2-0.238^2}$$

$$=\pm 0.129.$$

The observing error, therefore, exceeds the triangle error, and shows that the latter cannot be neglected in the true valuation of the respective weights.

The final value of the probable error (r) of any direction in the quadrilateral, we obtain, therefore, by combining the observing and the triangle error, the latter being constant. We then have

$$r=\sqrt{r_1^2+r_\Delta^2}$$

where $r_\Delta^2=0.0167$

The final weight of each direction is found from the equation

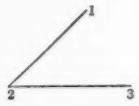
$$p=\frac{1}{r^2}$$

They are as follows:

Direction.	Recip. Wt.	Direction.	Recip. Wt.	Direction.	Recip. Wt.	Direction.	Recip. Wt.
12	.046	12	.046	23	.126	31	.101
14	70	13	.084	24	113	34	.154
21	169	21	.169	32	210	41	.074
24	113	23	.126	34	154	43	.107
41	74	31	.101	43	202	13	.084
42	202	32	.210	43	107	14	.070

THE GENERAL ADJUSTMENT.

74. *Notation.** An angle being considered as a difference of two directions may be expressed in terms of these directions.



Thus the angle 1 2 3 may be denoted

$$-\tfrac{1}{2}+\tfrac{3}{2}$$

21, 23 being the arms of the angle and 2 its vertex.

Corrections to a direction are indicated by being included in parentheses; thus $(\frac{1}{2})$ denotes a correction to $\frac{1}{2}$, and the correction to the angle 1 2 3 would be denoted by

$$-(\tfrac{1}{2})+(\tfrac{3}{2}).$$

75. In our example the number of angle-equations is 3, and of side-equations 1. We have

Angle-Equations.

1. The triangle 421

$$\begin{array}{rrrrr} & ^\circ & ' & '' & \\ 124= & 21 & 58 & 17.404 & -\frac{4}{2}+\frac{1}{2} \\ 241= & 79 & 35 & 42.479 & -\frac{1}{4}+\frac{2}{4} \\ 412= & 78 & 26 & 10.030 & -\frac{2}{1}+\frac{4}{1} \\ \hline & 180 & 00 & 9.013 & \\ 180+E= & 180 & 00 & 10.773 & \\ \hline & & & 0= & -0.860-\frac{4}{2}+\frac{1}{2}-\frac{1}{4}+\frac{2}{4}-\frac{2}{1}+\frac{4}{1} \end{array}$$

Similarly from the triangles 341, 231

$$\begin{array}{rrrrr} & ^\circ & ' & '' & \\ 0= & & & -1.562 & -\frac{1}{4}+\frac{3}{4}-\frac{3}{1}+\frac{4}{1}-\frac{4}{3}+\frac{1}{3} \\ 0= & & & -0.494 & -\frac{1}{3}+\frac{2}{3}-\frac{2}{1}+\frac{3}{1}-\frac{3}{2}+\frac{1}{2} \end{array}$$

76. *The Side-Equation.*

Take the pole at 1, then

$$\frac{\text{sin. } 123}{\text{sin. } 132}\cdot\frac{\text{sin. } 134}{\text{sin. } 143}\cdot\frac{\text{sin. } 142}{\text{sin. } 124}=1$$

$$\begin{array}{rrrrr} & ^\circ & ' & '' & \\ \angle 123= & 54 & 06 & 29.197 & -\frac{3}{2}+\frac{1}{2} \\ \angle 134= & 27 & 01 & 48.843 & -\frac{4}{3}+\frac{1}{3} \\ \angle 142= & 79 & 35 & 42.479 & -\frac{1}{4}+\frac{2}{4} \\ \hline & & & & \\ 132= & 101 & 44 & 3.123 & -\frac{1}{3}+\frac{2}{3} \\ 143= & 98 & 41 & 43.926 & -\frac{3}{4}+\frac{1}{4} \\ 124= & 21 & 58 & 17.404 & -\frac{4}{2}+\frac{1}{2} \end{array}$$

$$\begin{array}{l} 9.9085518.108+15.237(-\frac{3}{2}+\frac{1}{2}) \\ 9.6574962.451+41.269(-\frac{4}{3}+\frac{1}{3}) \\ 9.9927991.509+\ 3.866(-\frac{1}{4}+\frac{2}{4}) \\ \qquad 472.068 \\ \qquad 472.917 \\ \qquad \overline{\qquad\qquad} \\ -0.849 \end{array}$$

$$\begin{array}{l} 9.9908277.554-\ 4.372(-\frac{1}{3}+\frac{2}{3}) \\ 9.9949791.653-\ 3.220(-\frac{1}{4}+\frac{3}{4}) \\ 9.5730403.710+52.188(-\frac{4}{2}+\frac{1}{2}) \\ \qquad 472.917 \end{array}$$

and the conditional equation, with the differences expressed in units of the fifth place of decimals is

$$\begin{array}{l} 0.33951(\frac{1}{2})+.15237(\frac{3}{2})-.36896(\frac{1}{3}) \\ \quad +.41269(\frac{4}{3})+.07086(\frac{1}{4})-.03866(\frac{2}{4}) \\ \quad -.04373(\frac{2}{3})-.03220(\frac{3}{4})-.52188(\frac{4}{3}) \\ \qquad\qquad +0.00849=0 \end{array}$$

77. If now I, II, III, IV, denote the correlates of these four condition-equations, we have the correlate-equations:

$\frac{1}{wt.}$	v	I.	II.	III.	IV.
0.046	$(\frac{2}{1})$	−1	0	−1	0
.084	$(\frac{3}{4})$	0	−1	+1	0
.0					
.070	$(\frac{4}{1})$	+1	+1	0	0
.169	$(\frac{1}{2})$	+1	0	+1	−1.8476
.126	$(\frac{3}{2})$	0	0	−1	0.7618
.113	$(\frac{4}{2})$	−1	0	0	2.6094
.101	$(\frac{1}{3})$	0	+1	−1	1.8448
.210	$(\frac{2}{3})$	0	0	+1	0.2187
.154	$(\frac{4}{3})$	0	−1	0	2.0635
.074	$(\frac{1}{4})$	−1	−1	0	0.3543
.202	$(\frac{2}{4})$	+1	0	0	0.1933
0.107	$(\frac{3}{4})$	0	+1	0	0.1610

The normal equations may now be formed and solved as in Chauvenet, four places of decimals being retained.

The values of the conditions resulting from the solution of the normal equations when applied to the locally adjusted values give the finally adjusted values of the angles.

The triangle sides are completed from the formulæ

$$a=c.\ \text{sin. } (A-\frac{\varepsilon}{3}).\quad \text{cosec.}(C-\frac{\varepsilon}{3})$$

$$b=c\ \text{sin. } (B-\frac{\varepsilon}{3})\quad \text{cosec.}(C-\frac{\varepsilon}{3})$$

the base c being known.

* Compare Gerling's *Ausgleichungsrechnung.*

* See Reports for 1865 and 1878.

78. *The Precision determination.**

The strict application of the method of least squares to the computation of the probable errors of the adjusted distances is held to be too laborious, and accordingly only an approximate method is employed.

If the triangulation consists of a single chain of triangles, the computation of probable error is carried through the sides of the different triangles between the first and concluding lines. The triangles are independent of one another, and the p. e. r_a of a side a, b being the base is found from

$$r_a = a.\ r \sin. 1'' \sqrt{\tfrac{2}{3}[\cot.^2 A + \cot.^2 B + \cot. A \cot. B]}$$

where r is the average p. e. of an angle as deduced from the corrections to the observed angles, or the difference Δ between observed and adjusted angles. r may also be deduced approximately from the average error of closing of the triangles; also, from

$$r = .6745 \sqrt{\frac{[\Delta^2]}{3n}}$$

where n is the number of triangles, or better, from

$$r = .6745 \sqrt{2 \frac{[v^2]}{n_c}}$$

where n_c is the number of conditional equations and $[v^2]$ the sum of the squares of the direction-corrections.

If the triangulation consists of a chain of quadrilaterals, the p. e. of the last side as found above would need to be divided by $\sqrt{2}$, since we can arrive at that side through two different sets of triangles; and generally "it will be admissible to suppose this combination factor to be proportional to the square root of the fraction (f), number of angles of triangles used in computing r_a divided by number of angles in figure."

In the older work the formula

$$r_a = r\ a \sin. 1'' \sqrt{[\cot.^2 A \cot.^2 B]}$$

which is the same as the one used on the Lake Survey, was employed.

The p. e. found from this formula is too large, because it supposes only two of the angles of a triangle measured instead of three.

The result may be increased by the additional p. e. $\frac{a}{b} . r_b$, which is the effect of the p. e. of the measure of the base b. We have thus finally

$$r_1 = \sqrt{f . r_a^2 + \frac{a^2}{b^2} . r_b^2}.$$

THE HANSEN-SCHREIBER METHOD OF ADJUSTMENT.

79. The general forms that have been given in the articles preceding for the adjustment of a triangulation, though symmetrical and elegant, are quite complex. A method which shall give a marked diminution of work in the reduction without any loss of rigor in the results is a desideratum. Of those proposed I shall notice only that now carried out on the Prussian Land Survey. This method involves processes due to Gauss, Bessel and Hansen.

80. First, a few words as to the measurement of horizontal angles. Gauss, the father of modern geodesy, in his triangulation of Hanover, measured each angle independently of every other, but left no very full account of his modes of procedure. He used a repeating theodolite. The introduction and successful use of non-repeating theodolites by Struve in the measurement of the Russian arc of the meridian, led to the gradual abandonment of repeating theodolites, and consequently drew away attention from Gauss' methods. Through the influence and example of Bessel, the method of "arcs," as developed by him in the *Gradmessung in Ostpreussen*, has been generally followed in primary surveys since his time as being apparently better adapted to the non-repeating theodolite. But for many reasons which every observer must finally discover for himself, as he gains in experience, the conclusion is inevitable that the method of independent angles is the better of the two, even with a non-repeating theodolite.

The points to be considered are mainly accuracy of results; and secondarily, economy of time in making and reducing the observations. When we consider twist of station occupied, the different lengths of lines sighted over, the interruptions that

may occur in the course of reading an arc, the more uniform light that may always be had when the number of directions is small, the greater certainty of eliminating horizontal refraction, and the thoroughness with which periodic error of graduation of the limb of the theodolite can be eliminated, we must conclude that greater precision is to be attained by the method of independent angles. Even Andrae, the author of the most important contributions to the method of directions since Bessel, and who has used this method in the triangulation of Denmark, acknowledges that, "in place of observations of directions in arcs, it is preferable to return to the old method of Gauss of measuring angles." (*Verhandlungen der Europäischen Gradmessung*, 1878.)

As regards the cost it must be acknowledged, that for an equal number of results, leaving quality out of account, the method of arcs has the advantage. Nowadays, however, when facilities exist for measuring angles by night as well as by day, there is less delay in waiting for suitable conditions than when day-work alone had to be depended on. Taking this into account the difference in cost would not be great in any case, more especially as a triangulation party is never a very large one.

81. When the angles are measured independently, the observations may be so arranged as to make the reduction very systematic and easy, as will be shown presently.

This, indeed, is also *possible* with the method of arcs. For if the observations are continuous and every direction is equally well measured in every arc, then

$$p_\cdot = p_1 = \ldots = 1 = p' = p'' = \ldots .$$

and $\therefore$

$$[p'] = [p''] = \ldots\ldots\ldots = n_a$$
$$[p_1] = [p_2] = \ldots\ldots\ldots = n_s$$

The normal-equations (92) become, remembering that $[m'] = o$,

$$\left(n_a - \frac{n_a}{n_s}\right)x - \frac{n_a}{n_s} \cdot y - \ldots = [m''] - \frac{[m]}{n_s}$$
$$+\left(n_a - \frac{n_a}{n_s}\right)y - \ldots = [m'''] - \frac{[m]}{n_s}$$
$$\ldots\ldots\ldots\ldots\ldots$$

By addition the number of equations being $n_s - 1$

$$\frac{n_a}{n_s}(x = y + + \ldots .) = \frac{[m]}{n_s}$$

Hence the normal-equations reduce to

$$n_a \cdot x = [m'']$$
$$n_a \cdot y = [m'''] \qquad (102).$$

and $\therefore$ the corrections are known.

Also (93) reduces to

$$\mu^2 = \frac{[vv]}{(n_a - 1)(n_s - 1)} \quad . \quad . \quad (103).$$

This method of reading each arc completely is not *practicable* except when the number of arcs is very small, as the time required would be too great to wait for suitable conditions. Besides, the values of the resulting angles would not be so accurate as if the other method of reading had been employed.

82. In order to diminish the labor of reducing the triangulation the angles must be measured according to some regular form. If measured differently at different stations there is a new problem to be solved for each.

To Gauss and Gerling is attributed the plan of measuring every angle at a station between every two directions. There are many advantages in doing this.

Thus let 0 be the station occupied, and 1, 2, 3, 4, the stations sighted at in order round the horizon; then the angles measured would be

102		
103	203	
104	204	304

Take the first three as independent unknowns.

Then, since

most prob. value = measured value + resid. v

if (A), (B), (C), be the corrections to the assumed approximate values to make the most probable values and (1, 2), and (1, 3) . . the corrections to these approximate values to make the measured values we have the observation-equations

$$\begin{aligned}
&(A) && -(1.2) = v_{1.2}\\
&(B) && -(1.3) = v_{1.3}\\
&(C) && -(1.4) = v_{1.4}\\
-(A) + (B) && && -(2.3) = v_{2.3} \qquad (104).\\
-(A) && + (C) && -(2.4) = v_{2.4}\\
&-(B) && + (C) && -(3.4) = v_{3.4}
\end{aligned}$$

and the normal-equations

$$\begin{aligned} 3(A)-(B)+(C)&=[al]\\ 3(B)-(C)&=[bl]\\ 3(C)&=[cl] \end{aligned} \qquad (105).$$

where

$$\begin{aligned} [al]&=(12)-(23)-(24)\\ [bl]&=(13)+(23)-(34)\\ [cl]&=(14)+(24)+(34) \end{aligned} \qquad (106).$$

Solving these equations (A). (B) (C) are found.

83. So far the solution is that already given in Art. 35, and if the station adjustment only were required we could stop here, as the above equations being symmetrical in form are easily handled.

But in passing on to the net adjustment a very considerable reduction of work can be effected by finding the corrections to the directions instead of to the angles. Thus if X, A, B, C, denote the corrections to the four directions 1, 2, 3, 4, then since A−X, B−X, C−X correspond to the (A), (B), (C), above, the observation-equations may be written

$$\begin{aligned} -X+A\qquad\quad -(12)&=v_{1.2}\\ -X\quad +B\quad -(13)&=v_{1.3}\\ -X\qquad +C-(14)&=v_{1.4}\\ -A+B\quad -(23)&=v_{2.3}\\ -A\quad +C-(24)&=v_{2.4}\\ -B+C-(34)&=v_{3.4} \end{aligned} \qquad (107).$$

and the normal-equations,

$$\begin{aligned} 3X-A-B-C&=-[al]-[bl]-[cl]\\ 3A-B-C&=[al]\\ 2B-C&=[bl]\\ 3C&=[cl] \end{aligned}$$

By addition of these equations there results:

$$0=0$$

and ∴ the unknowns cannot be found without another relation between X, A, B, C. The reason of this is that directions are nothing but the angles which the rays make with some common zero ray whose position is unknown, and which therefore may be taken arbitrarily. To solve the above equations it will be most convenient to make the arbitrary assumption

$$X+A+B+C=0 \quad . \quad . \qquad (109).$$

and the normal-equations reduce to

$$\begin{aligned} 4X&=-[al]-[bl]-[cl]\\ 4A&=[al]\\ 4B&=[bl]\\ 4C&=[cl] \end{aligned} \qquad (110).$$

that is

$$\begin{aligned} 4X&=-(12)-(13)-(14)\\ 4A&=(12)-(23)-(24)\\ 4B&=(13)+(23)-(34)\\ 4C&=(14)+(24)+(34) \end{aligned} \qquad (111).$$

which can be very conveniently computed from the scheme due to Col. Schreiber:

1	2	3	4	Sums.
	(12)	(13)	(14)	σ_1
		(23)	(24)	σ_2
			(34)	σ_3
	s_2	s_3	s_4	
$-\sigma_1$	$-\sigma_1$	$-\sigma_3$		
$[al]$	$[bl]$	$[cl]$	$[dl]$	

84. The general adjustment due to the side and angle-equations in the net is equally simple.

The weight-equations, corresponding to equations (42), are:

$$\begin{aligned} 4[aa]&=1\\ 4[a\beta]&=0\\ 4[a\gamma]&=0\\ 4[a\delta]&=0 \end{aligned} \qquad (112).$$

Similarly for the other unknowns. Hence in this case

$$\begin{aligned} [aa]=[\beta\beta]=[\gamma\gamma]&=\tfrac{1}{4}\\ [a\beta]=[a\gamma]=\ .\ .\ &=0 \end{aligned}$$

and since

$$\begin{aligned} [\overline{aA}]&=[aa]\\ [\overline{aB}]&=[ab]\\ &\cdots\cdots \end{aligned}$$

the net correlate normal-equations (47) become

$$\begin{aligned} [aa]I+[ab]II+[ac]III&=l_a'\\ [bb]II+[bc]III&=l_a''\\ [cc]III&=l_a''' \end{aligned} \qquad (113).$$

just as if formed from

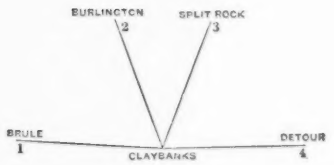
$$\begin{aligned} a'.1+b'.II+c'.III\\ +b''.II+c''.III\\ +c'''.III \end{aligned}$$

Also the equations (43) from which the corrections (1) . (2). . . . are found, reduce to the simple form

$$\begin{aligned} (1)&=a'.I+b'.II+\ldots.\\ (2)&=a''.I+b''.II\\ &\cdots\cdots\cdots \end{aligned} \qquad (114).$$

85. *Example of Station Adjustment.* Given at station Claybanks in the triangulation of Lake Superior

	°	′	″
12=	61	33	23.42
13=100	23	02.38	
14=170	02	03.37	
23=	38	49	38.70
24=108	28	38.98	
34=	69	38	60.47



Assumed approximate angles:

	°	′	″
Brule.....	0	00	00+A
Burlington..	61	33	23+B
Split Rock..	100	23	02+C
Detour.....	170	02	03+D

1	2	3	4	Sums.
	+0.42	+0.38	+0.37	+1.17
		−0 30	−1.02	−1.32
			−0.53	−0.53
	+0.42	+0.08	−1.18	
−1.17	+1.32	+0.53		
−1.17	+1.74	+0.61	−1.18	

″

$\therefore$ A = −0.29½
B = +0.43½
C = +0.15½
D = −0.29½

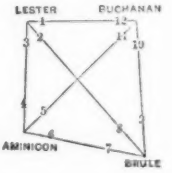
and the adjusted angles,

	°	′	″
12=	61	33	23.73
13=100	23	02.45	
14=170	02	03.00	
23=	38	49	38.72
24=108	28	39.27	
34=	69	38	60.55

86. *Example of Station and Net Adjustment.*

The quadrilateral Aminicon, Brule, Buchanan, Lester.

	Direction.	Measured angle. ° ′ ″	Locally adjusted angle. ″
At Lester....	2−1	51 00 39.77	40.00
	3−1	92 43 49.42	49.20
	3−2	41 43 08.97	9.20
At Aminicon.	5−4	37 47 05.51	5.17
	6−4	97 51 03.11	3.45
	6−5	60 03 58.62	58.28
At Brule.....	8−7	40 25 47.49	47.36
	9−7	71 58 27.31	27.45
	9−8	31 32 40.22	40.19
At Buchanan.	11−10	47 57 36.25	36 21
	12−10	97 26 41.29	41.32
	12−11	49 29 05.15	5.11



The condition-equations formed in the usual way from the triangles Lester, Buchanan, Brule, (ε=1″.19); Lester, Aminicon, Brule, (ε=1″.19); Aminicon, Buchanan, Brule, (ε=1″.37); and from the quadrilateral (pole at Lester) are

$$-(1)+(2)-(8)$$
$$-(2)+(3)-(4)$$
$$-(5)+(6)-(7)$$
$$+3.01(4)-2.72(5)-0.29(6)$$
$$+(9)-(10)+(12)$$
$$+(6)-(7)+(8)$$
$$+(9)-(10)+(11)$$
$$+2.47(7)-5.90(8)+3.43(9)$$
$$+0''.22=0$$
$$+1.18=0$$
$$-0.57=0$$
$$-0.28(10)-1.80(11)+2.08(12)+1.60=0$$

Auxiliary table for forming the normal-equations:

I.	II.	III.	IV.
−1			
+1	−1		
	+1		
	−1		+3.01
		−1	−2.72
	+1	+1	− .29
	−1	−1	+2.47
−1	+1		−5.90
+1		+1	+3.43
−1		−1	−0.28
		+1	−1.80
+1			+2.08

The normal-equations:

I.	II.	III.	IV.
+6	−2	+2	+11.69 = +0.22
	+6	+2	−11.67 = −1.18
		+6	+ 1.87 = −0.57
			+76.86 = +1.60

$\therefore$ I = +0.0095 III = +0.0181
II = +0.2123 IV = +0.0095

The corrections:

(1) = −0.01″	(7) = −0.21″
(2) = −0.20	(8) = +0.15
(3) = +0.21	(9) = +0.06
(4) = −0.18	(10) = −0.03
(5) = −0.04	(11) = 0.00
(6) = +0.23	(12) = +0.03

Approximate Method of Adjustment.

87. In an extensive triangulation the form given above is very convenient and gives accurate results. In a chain of central polygons the Schleiermacher solution * may be used to advantage. In this country, however, where chains of triangles and quadrilaterals of the ordinary form have have been more common than central polygons and are likely to be so on account of greater cheapness, a rapid and nearly exact rule for a reduction of this kind will now be given. This will be of special advantage in surveys of rivers where a long string of triangles and quadrilaterals is unavoidable. In more complicated cases, such as triangulating for finding the axis of a tunnel, a complex net is pretty certain to be the form and for the planning and adjustment of this there is nothing better than that given in Arts. 82–86. As an example of tunnel adjustment, the discussion by Koppe of the triangulation for finding the axis of the St. Gothard Tunnel may be cited. (*Zeitschr. für Vermess.* 1875.)

It shall be my purpose in this chapter to derive rules for solving a triangulation chain of a purely mechanical kind requiring no knowledge of least squares in order to apply them.

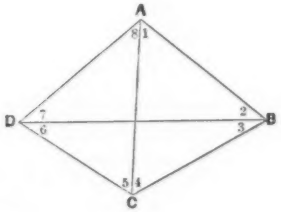
The principle underlying the whole process is that of successive approximation as explained in Art. 27. After an adjustment has been made for one set of conditions, the resulting angles are introduced as measured angles into the adjustment arising from the next set of conditions. The steps are as follows:

(*a*) Adjustment for sum-angles. Rule in Art. 35.

(*b*) Adjustment for closure of the horizon. Rule in Art. 35.

(*c*) Adjustment of a single triangle, all of the angles being measured.

(*d*) Adjustment of a single quadrilateral ABCD, all of the eight angles being measured. A rule for this may be derived as follows: With the usual notation the condition-equations may be written in general terms in two sets.



(1.) The angle-equations from the triangles ABC, BCD, CDA.

$$v_1+v_2+v_3+v_4=l_1$$
$$v_3+v_4+v_5+v_6=l_2$$
$$v_5+v_6+v_7+v_8=l_3$$

(2). The side-equation pole C,

$$a_1v_1+a_2v_2+a_3v_3+a_4v_4+a_5v_5 + a_6v_6+a_7v_7+a_8v_8=l_4$$

As in Bessel's form we break the solution into two parts.

The correlate-equations are:

$$\begin{aligned} k_1 \qquad\qquad &+v_1=0\\ k_1 \qquad\qquad &+v_2=0\\ k_1+k_2 \qquad\quad &+v_3=0\\ k_1+k_2 \qquad\quad &+v_4=0\\ k_2+k_3 \qquad\quad &+v_5=0\\ k_2+k_3 \qquad\quad &+v_6=0\\ k_3 \qquad\qquad &+v_7=0\\ k_3 \qquad\qquad &+v_8=0 \end{aligned}$$

and the normal-equations:

$$\begin{aligned} 4k_1+2k_2 \qquad\quad &+l_1=0\\ 2k_1+4k_2+2k_3 &+l_2=0\\ 2k_2+4k_3 &+l_3=0 \end{aligned}$$

Solving these equations there result:

$$k_1=\tfrac{1}{8}(-3l_1+2l_2-\ l_3)$$
$$k_2=\tfrac{1}{8}(+2l_1-4l_2+2l_3)$$
$$k_3=\tfrac{1}{8}(-\ l_1+2l_2-3l_3)$$

* See Fischer's "Geodäsie," Part III.

Substituting these values in the correlate-equations, and

$$\begin{aligned} v_1=v_2&=\tfrac{1}{8}(+3l_1-2l_2+\ l_3\\ v_3=v_4&=\tfrac{1}{8}(+\ l_1+2l_2-\ l_3\\ v_5=v_6&=\tfrac{1}{8}(-\ l_1+2l_2+\ l_3\\ v_7=v_8&=\tfrac{1}{8}(+\ l_1-2l_2+3l_3 \end{aligned}$$

which may be written:

$$\begin{aligned} v_1=v_2&=\tfrac{1}{4}l_1-\tfrac{1}{4}(l_3-\tfrac{1}{2}l_2-\tfrac{1}{2}l_1)\\ v_3=v_4&=\tfrac{1}{4}l_1+\tfrac{1}{4}(l_3-\tfrac{1}{2}l_2-\tfrac{1}{2}l_1)\\ v_5=v_6&=\tfrac{1}{4}l_2+\tfrac{1}{4}(l_3-\tfrac{1}{2}l_2-\tfrac{1}{2}l_1)\\ v_7=v_8&=\tfrac{1}{4}l_3-\tfrac{1}{4}(l_3-\tfrac{1}{2}l_2-\tfrac{1}{2}l_1) \end{aligned}$$

whence follows at once the convenient rule for adjusting the quadrilateral so far as the angle-equations are concerned.

1. Write the measured angles in order of azimuth in two sets of four each, the first set being the angles of ABC and the second those of CDA.
2. Adjust the angles of each set by $\frac{1}{4}$ of the difference of this sum from 180° + excess of triangle, arranging the adjusted angles in two columns so that the first column will show the angles of ABD and the second those of CBD.
3. Adjust the first column by $\frac{1}{4}$ of the difference of its sum from 180° + excess of triangle and apply the same correction to the second column with the opposite sign.

90. If now $v_1', v_2' \ldots$ denote the further corrections arising from the side-equation, the condition-equations become:

$$\begin{aligned} v_1'+v_2'+v_3'+v_4'&=0\\ v_3'+v_4'+v_5'+v_6'&=0\\ v_5'+v_6'+v_7'+v_8'&=0\\ [av']&=l_4' \end{aligned}$$

with

$$[v'v']=\text{a minimum.}$$

The solution could be carried out by correlates as usual; but by the following artifice the work is much shortened. By writing the corrections in Hansen's form:

$$\begin{aligned} v_1'&=+v+v' & v_5'&=+v+v'''\\ v_2'&=+v-v' & v_6'&=+v-v'''\\ v_3'&=-v+v'' & v_7'&=-v+v''''\\ v_4'&=-v-v'' & v_8'&=-v-v'''' \end{aligned}$$

the first three condition-equations become 0=0 identically, and we have therefore to deal only with the single condition-equation:

$$\begin{aligned} &a_1(v+v')+a_2(v-v')+a_3(-v+v'')\\ &\quad+a_4(-v-v'')+a_5(v+v''')+a_6(v-v''')\\ &\qquad+a_7(-v+v'''')+a_8(-v-v'''')=l_4' \end{aligned}$$

or

$$\begin{aligned} &(a_1+a_2-a_3-a_4+a_5+a_6-a_7-a_8)v\\ &\quad+(a_1-a_2)v'+(a_3-a_4)v''+(a_5-a_6)v'''\\ &\qquad+(a_7-a_8)v''''=l_4' \end{aligned}$$

The correlate-equations are:

$$\begin{aligned} k(a_1+a_2-a_3-a_4+a_5+a_6-a_7-a_8)&=4v\\ k(a_1-a_2)&=v'\\ k(a_3-a_4)&=v''\\ k(a_5-a_6)&=v'''\\ k(a_7-a_8)&=v'''' \end{aligned}$$

Hence, equating the values of k,

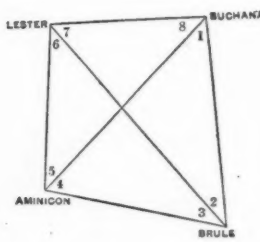
$$\begin{aligned} \frac{v}{\tfrac{1}{4}(a_1+a_2-a_3-a_4+a_5+a_6-a_7-a_8)}&=\frac{v'}{a_1-a_2}\\ &=\frac{v''}{a_3-a_4}\\ &=\frac{v'''}{a_5-a_6}\\ &=\frac{v''''}{a_7-a_8} \end{aligned}$$

$$=\frac{l_4'}{\tfrac{1}{4}(a_1+a_2-a_3-a_4+a_5+a_6-a_7-a_8)^2+(a_1-a_2)^2+(a_3-a_4)^2+(a_5-a_6)^2+(a_7-a_8)^2}$$

and, therefore, the corrections are known.

This adjustment is in all respects rigid.

91. *Example.*—Given in the quadrilateral Buchanan, Brule, Aminicon, Lester, the measured angles:



	°	′	″
1=	47	57	36.25
2=	31	32	40.22
3=	40	25	47.49
4=	60	03	58.62
5=	37	47	05.51
6=	41	43	08.97
7=	51	00	39.77
8=	49	29	05.15

ADJUSTMENT.

	Measured Angles.	First Correction.	Angles.	Log Sines.		Diff. 1″		Sums.	Squares
	° ′ ″	″	″						
1	47 57 36.25	35.95	35.85	9.8708000		+1.90			
2	31 32 40.22	39.92 ″	39.82		9.7186338		+3.43	5.33	28.41
3	40 25 47.49	47.19	47.29	9.8119206			+2.47		
4	60 03 58.62	58.32	58.42		9.9378201	+1.21		3.68	13.54
	180 00 02.58								
	180 00 01.37	$=180+\varepsilon$							
	4)−1.21								
	−0.30								
5	37 47 05.51	05.91	06.01	9.7872480		+2.72		5.08	25.81
6	41 43 08.97	09.37	09.47		9.8231362		+2.36		
7	51 00 38.77	40.17	40 07	9.8905709			+1.70	3.50	12.25
8	49 29 05.15	05.55	05.45		9.8809474	+1.80			
	179 59 59.40	1.59		395	375	+7.63	+9.96		
	180 00 01.01	1.19		375			+7.63		
	4)+1.61	4)−0.40		20			4)+2.33	0.58	0.34
	+0.40	−0.10					+0.58		80.35

Hence

$$\frac{v}{0.58}-\frac{v^{I}}{5.33}=\frac{v^{II}}{3.68}=\frac{v^{III}}{5.08}=\frac{v^{IV}}{3.50}=\frac{20}{80.35}$$

and

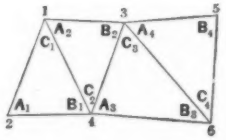
$$\begin{aligned} v &= 0.144 & +v+v' &= +1.47 \\ v^{I} &= 1.327 & +v-v' &= -1.18 \\ v^{II} &= 0.916 & -v+v^{II} &= +0.77 \\ v^{III} &= 1.264 & -v-v^{II} &= -1.06 \\ v^{IV} &= 0.871 & +v+v^{III} &= +1.41 \\ & & +v-v^{III} &= -1.12 \\ & & -v+v^{IV} &= +0.73 \\ & & -v-v^{IV} &= -1.02 \end{aligned}$$

Corrected Angles.

°	′	″
47	57	37.32
31	32	38.64
40	25	48.06
60	03	57.36
37	47	07.42
41	43	08.35
51	00	40.80
49	29	04.43

ADJUSTMENT OF A TRIANGULATION FOR AZIMUTH AND BASE LINE CONDITIONS.*

So far we have considered the adjustment of a triangulation with reference to only one measured base. But in an extended chain where bases are measured at intervals, it is important, especially in secondary and tertiary work, to adjust so as to allow the bases to exert their proper influence. Not only so, but if the bases are known in azimuth as well as in length, the intervening triangulation should be made to conform to this as well.



Let 12 and 56 be two bases connected by a net of triangles through intermediate stations 3, 4. The lengths and azimuths of 12 and 56 are assumed to be given correctly.

It is supposed that the net has been already adjusted for local conditions and for the geometrical conditions. Now, omitting all superfluous connections, we may reduce the net to a single chain of triangles. Let us then consider 12 and 56 as connected by a single chain of the best shaped triangles that can be selected from the system.

* See Andrae in *Verhandlungen der Europäischen Gradmessung*, 1877. *G. T. Survey of India*, Vol. 2.

When 56 is computed from 12, the computed value and observed value do not in general agree, and when the azimuth of 56 is computed from that of 12 the computed and known values do not in general agree. We therefore still farther adjust the angles of the triangles so that these discrepancies disappear and that the angles of the triangles may still satisfy the conditions of closure. This is strictly in accordance with the general theorem (27).

We shall adjust for the discrepancies one at a time.

(1.) *Correction for discrepancy in azimuth.*

Let A_1, A_2....be the angles opposite to the sides of continuation.

B_1, B_2....be the angles opposite the bases in order of computation.

C_1, C_2....be the angles opposite to the flank sides.

Let (A_1),(B_1)....denote the corrections to A_1, B_1.....

Now reckoning azimuth from the south round by the west, and passing from the base 12 along the sides 13-35,....the excess l'_2 of the observed over the computed value of the azimuth of the last line of continuation 56 is

$$-(C_1)-(A_2)-(B_2)-(C_2)-(A_4)-(B_4)$$

and passing through the sides 24, 46, . . to the same line l_2'' is

$$(A_1)+(B_1)+(C_2)+(A_3)+(B_3)+(C_4)$$

Hence by addition putting $l_2'+l_2''=2l_2$,

$$\begin{aligned} 2l_2=&(A_1)+(B_1)-(C_1)\\ &-(A_2)-(B_2)+(C_2)\\ &+(A_3)+(B_3)-(C_3)\\ &-(A_4)-(B_4)+(C_4) \end{aligned} \qquad (115).$$

the angles adjacent to the flank sides having the + sign and the angles opposite to the flank sides having the − sign on one side of the chain and the + on the other side.

We have also the conditions since the triangle close

$$\begin{aligned} &(A_1)+(B_1)+(C_1)=0\\ &(A_2)+(B_2)+(C_2)=0\\ &(A_3)+(B_3)+(C_3)=0\\ &(A_4)+(B_4)+(C_4)=0 \end{aligned} \qquad (116).$$

all being subject to the relation

$$(A)^2+(B)^2+(C)^2+\ldots.=\text{a minimum.}$$

Calling k_1, k_2....the correlates of equations (116) and k that of equation (115) we have

$$\begin{aligned} &k_1+k=(A_1) \qquad k_2-k=(A_2)\\ &k_1+k=(B_1) \qquad k_2-k=(B_2)\\ &k_1-k=(C_1) \qquad k_2+k=(C_2)\\ &k_3+k=(A_3) \qquad k_4-k=(A_4)\\ &k_3+k=(B_3) \qquad k_4-k=(B_4)\\ &k_3-k=(C_3) \qquad k_4+k=(C_4) \end{aligned}$$

and the normal-equations

$$\begin{aligned} 3k_1 \qquad\qquad &+k=0\\ 3k_2 \qquad\qquad &-k=0\\ 3k_3 \qquad\qquad &+k=0\\ 3k_4 \qquad\qquad &-k=0\\ k_1-k_2+k_3-k_4+12k=&2l_2 \end{aligned}$$

Hence solving we should have for n triangles

$$\begin{aligned} 2\tfrac{2}{3}\,n\,k=&(A_1)+(B_1)-(C_1)\\ &-(A_2)-(B_2)+(C_2)\\ &\ldots\ldots\ldots\ldots\\ =&2l_2 \end{aligned}$$

and

$$k=\frac{3l_2}{4n}$$

also

$$k_1=-k_2=\ldots.=-\frac{1}{4n}.l_2$$

$\therefore$

$$(A_1)=+\frac{1}{2n}.l_2 \qquad (A_2)=-\frac{1}{2n}.l_2\ldots\ldots$$

$$(B_1)=+\frac{1}{2n}.l_2 \qquad (B_2)=-\frac{1}{2n}.l_2\ldots\ldots$$

$$(C_1)=-\frac{1}{n}.l_2 \qquad (C_2)=-\frac{1}{n}.l_2\ldots\ldots$$

Hence the rule. *Divide the excess of the observed over the computed azimuth by the number of triangles, and apply one-half of this quantity to each of the angles adjacent to the flank sides on one side of the chain, and the total quantity with the sign changed to the third angle.* The signs are reversed for the angles on the other flank.

If the azimuth mark is not on a triangulation side, the line of azimuth may be swung on to such a side by adding the angle between the mark and the station.

If, again, the azimuth is observed at intermediate points and not at the ends of the system, a slight error may come into the base line triangles. The process can be repeated if the discrepancy is too great.

(2) *Correction for discrepancy in bases.*
Call the angles adjusted for local and general conditions and for discrepancy in azimuth $A_1', B_1', \ldots$. Computing 56 from 12 through the intervening triangles we find

$$\text{Computed } 56 = 12\frac{\sin. A_1'.\sin. A_2'\ldots.}{\sin. B_1'.\sin. B_2'\ldots.}$$

But the given value of 56 being the most probable value we have
Given 56

$$= 12\frac{\sin.[A_1'+(A_1')].\sin.[A_2'+(A_2')]\ldots.}{\sin.[B_1'+(B_1')].\sin.[B_2'+(B_2')]\ldots.}$$

If $\delta_A', \delta_A'', \ldots \delta_B', \delta_B'' \ldots$ are the tabular differences corresponding to 1″ for the angles $A_1, A_2, \ldots B_1, B_2, \ldots$ in a table of log. sines and

log. given base 56 − log computed base 56 $= l_b$

then

$$\delta_A'(A_1') + \delta_A''(A_2') + \ldots. - \delta_B'(B_1') - \delta_B''(B_2') - \ldots. = l_b$$

Also since the triangles close

$$(A_1') + (B_1') + (C_1') = 0$$
$$(A_2') + (B_2') + (C_2') = 0$$
$$\ldots\ldots\ldots\ldots$$

From these equations it is evident that the corrections to the angle C are small compared with those to A and B, and that they vanish when A = B. Hence, as we have assumed the triangles to be well shaped we may take

$$(C_1') = (C_2') = \ldots. = 0$$

The angle-equations then become

$$(A_1') + (B_1') = 0$$
$$(A_2') + (B_2') = 0$$
$$\ldots\ldots\ldots$$

and the correlate-equations

$$k' + k_0\delta_A = (A_1') \qquad k'' + k_0\delta_A'' = (A_2') \ldots.$$
$$k' - k_0\delta_B = (B_1') \qquad k'' - k_0\delta_B'' = (B_2') \ldots.$$

whence

$$(A_1') = -(B_1') = \frac{\delta_A' + \delta_B'}{2} k_0$$

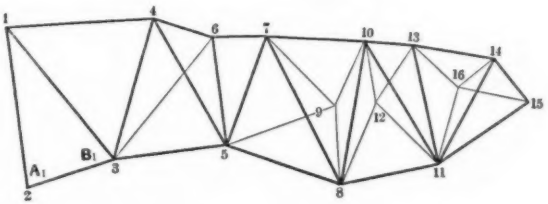
$$(A_2') = -(B_2') = \frac{\delta_A' + \delta_B'}{2} k_0$$

$$\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots$$

Substitute these values in the base line equation and

$$\frac{[(\delta_A + \delta_B)^2]}{2} . k_0 = l_b$$

Hence k_0 is known and therefore



Call $k', k'' \ldots$ the correlates of the angle-equations and k_0 that of the side-equation, then the correlate-equations are:

$$k' + k_0.\delta_A' = (A_1') \quad , \quad k'' + k_0\delta_A'' = (A_2') \ldots$$
$$k' - k_0.\delta_B' = (B_1') \quad , \quad k'' - k_0\delta_B'' = (B_2') \ldots$$
$$k' \qquad = (C_1') \quad , \quad k'' \qquad = (C_2') \ldots$$

Hence eliminating $k', k'', \ldots$ by combining these equations and the angle equations, then result

$$(A_1') = +\tfrac{1}{3}k_0(2\delta_A' + \delta_B')$$
$$(B_1') = -\tfrac{1}{3}k_0(\delta_A' + 2\delta_B')$$
$$(C_1') = -\tfrac{1}{3}k_0(\delta_A' - \delta_B')$$

$$(A_2') = +\tfrac{1}{3}(2\delta_A'' + \delta_B'') \ldots.$$
$$(B_2') = -\tfrac{1}{3}(\delta_A'' + 2\delta_B'') \ldots.$$
$$(C_2') = -\tfrac{1}{3}(\delta_A'' - \delta_B'') \ldots.$$

$$A_1' = -B_1' = \frac{\delta'_A + '\delta_B}{[(\delta_A + \delta_B)^2]} . l_b$$

$$A_2' = -B_2' = \frac{\delta''_A + \delta''_B}{[(\delta_A + \delta_B)^2]} . l_b$$

$$\ldots\ldots\ldots\ldots\ldots$$

that is, the corrections are known.

The corrections to the angles from the base line adjustment will not disturb the azimuth equation, since in each triangle

$$(A') + (B') - (C') = 0.$$

Hence it is not necessary to repeat the process.

94. *Example.*—The above sketch represents the secondary triangulation of

Long Island Sound.* It is joined to the primary triangulation at the lines 1-2, 14-15. These lines are assumed to remain unchanged in length and azimuth in the adjustment, and the secondary system is made to conform to them.

The main chain of triangles joining the two primary lines is indicated in the figure by heavy lines. The number of these triangles is 11. The system has been adjusted for local and geometrical conditions, and the resulting angles of these triangles are as follows:

	Angle.			ϵ	Log. Sin. Diff. 1″
	°	′	″	″	
A_1	82	49	19 25	3.66	0.27
B_1	64	55	40.32		0.99
C_1	32	15	04.09		
A_2	52	37	47 98	4.96	1.61
B_2	74	10	50.56		0.60
C_2	53	11	26 42		
A_3	69	20	16.03	3.52	.79
B_3	67	43	43.33		.86
C_3	42	56	04.16		
A_4	39	31	29.66	1.31	2.55
B_4	120	36	36.11		−1.25
C_4	19	51	55.54		
A_5	83	19	31.08	1.41	0.25
B_5	68	57	47.75		0.81
C_5	27	42	42.58		
A_6	87	25	26.66	3.47	0.09
B_6	44	10	45.25		2.17
C_6	48	23	51.56		
A_7	58	25	08.08	3.63	1.29
B_7	82	16	16.16		0.29
C_7	39	18	39.39		
A_8	69	12	06.02	3.03	0.80
B_8	71	58	54.18		0.68
C_8	38	49	02.83		
A_9	59	18	26.33	1.33	1.25
B_9	102	08	26.94		−0.45
C_9	18	33	08.05		
A_{10}	67	38	52.77	2.41	0.87
B_{10}	70	26	23.26		0.75
C_{10}	41	54	46.38		
A_{11}	29	43	55.99	2.13	3.69
B_{11}	55	23	09.36		1.45
C_{11}	94	52	56.78		

M.

Distance, 1- 2=49641.82
" 14-15=22523.20

* Coast Survey Report, 1868.

(1) *The Adjustment for Azimuth.*

After the local and general adjustments were made, a geodetic computation for latitude, longitude, and azimuth was carried through from the line 1-2 to 14-15. It was found that

observed azimuth of 14-15
−computed az. of do=−2″.93

Hence the corrections to the angles of the triangles for this discrepancy in azimuth are for the

first triangle $A_1=-0.13$
$B_1=-0.13$
$C_1=+0.27$
second triangle $A_2=+0.13$
$B_2=+0.13$
$C_2=-0.27$

and so on.

(2) *The Adjustment of Base-lines.*

We have

$$\frac{14-15}{1-2}=\frac{\sin.\,[A_1+(A_1)]}{\sin.\,[B_1+(B_1)]}\cdot\frac{\sin.\,[A_2+(A_2)]}{\sin.\,[B_2+(B_2)]}\cdot\cdot\cdot\cdot\cdot\frac{\sin.\,[A_{11}+(A_{11})]}{\sin.\,[B_{11}+(B_{11})]}\cdot$$

or substituting the values of the angles and reducing to the linear form

$$.27(A_1)-0.99(B_1)+1.61(A_2)-0.60(B_2)+\ldots.=0.40$$

Hence

$$[(\delta_A+\delta_B)^2]=51.48$$

and

$$(A_1)=-(B')=\frac{1.26}{51.48}\times 0.40=0.00$$

$$(A_2)=-(B_2)=\frac{2.21}{51.48}\times 0.40=0.00$$

.....

Applying the corrections for the azimuth and side-equations to the already adjusted values we have the final angles.

95. A somewhat different method was used on the Coast Survey in the adjustment of the system of secondary triangulation just given.*

The adjustment consists in an exact conformity to the primary triangulation, to which this secondary series is joined at each end, and to adjust the geometrical conditions in the secondary triangulation. The results deduced cannot differ

* C. S. Report, 1868.

from the most probable by more than the p. e. The successive steps are

1. The measured angles at each station are adjusted if necessary, weights being taken into account. At the primary stations 1, 2, 14, 15, the primary directions remain unchanged, and the secondary directions must be made to conform to them.
2. The condition-equations, 17 angle and 7 side-equations, are formed and solved. The triangle sides are then computed, starting from one of the primary lines, as Ruland-Tashua (1—2), from which the latitude, longitude and azimuth of the whole is computed. This gives

log. McSparran E.Rock (14–15) 4.3526260
log. do. from pri. triangulation. 4.3526300

	°	′	″
Az. of line McSparran-E.Rock	282	45	31.382
do. from pri. triangulation.			28.446
Diff. at McSparran....			−2.936
Diff. at E. Rock.....			−2.937
Mean.....			−2.931

3. We have now to adjust for the difference in length of the two values of the side McSparran—E. Rock by adding the equation

$$\frac{14.15}{12.1} = \frac{\text{sin. 123. sin. 413....}}{\text{sin. 231. sin. 341....}}$$

and to adjust for the difference in azimuth by adding the azimuth-equation.

$$\binom{3}{2}-\binom{2}{3}+\binom{5}{3}-\binom{3}{5}+\binom{8}{5}-\binom{5}{8}+\binom{11}{8}$$
$$-\binom{8}{11}+\binom{15}{11}-\binom{11}{15}+2''.93=0$$

to the previously-found condition-equations. This makes 26 condition-equations which are again solved and the triangle sides, the latitudes, longitudes and azimuths of all the points recomputed. The secondary line, McSparran—E. Rock, will now of course agree in length and direction with the primary line, but the extremities do not coincide.

4. The small residual differences in latitude (=−0″.007) and in longitude =(−0″.038) are next corrected proportionally to the distance from Tashua, and thus the correct latitude and longitude of each point found.

5. Each station is next reduced to center, the triangle sides recomputed, and a third latitude, longitude and azimuth computation carried through. No contradiction will now appear among the results.

AMERICAN PRACTICE IN WARMING BUILDINGS BY STEAM.

By the late ROBERT BRIGGS, M. Inst. C. E.

From Proceedings of the Institution of Civil Engineers.

II.

Circulation of Steam for Warming.—There are two methods of warming with steam, one with live steam direct from the boiler, and the other with exhaust steam. These two are frequently carried out in combination, and in fact generally so where exhaust steam is used at all for warming. The circulation or distribution of the steam through the warming pipes is effected in an almost unlimited variety of ways, each possessing advantages for special cases. The cause producing the circulation throughout the pipes of the warming apparatus is solely the difference of pressure which results from the more or less rapid condensation of the steam in contact with the radiating surfaces; a partial vacuum of greater or less amount is thereby formed within the radiating portions of the apparatus, and the column of steam or of water equivalent to this diminution of pressure constitutes the effective head producing the flow of steam from the boiler, while the return current of condensed water is determined by the downward inclination of the pipes for the return course.

When using live steam direct from the boiler, the system of what is called closed circulation is carried out either with separate supply and return mains, both of which extend to the furthest distance to which the heat has to be distributed; or else with a single main, which answers at

once for both the supply and the return, either with or without a longitudinal partition inside it for separating the outward current of steam supply from the return current of condensed water. In what is called the system of open circulation, a supply main conveys the steam to the radiating surfaces, whence a return main conducts the condensed water either into an open tank for feeding the boiler, or into a drain to run to waste, the boiler being then fed from some other source; in either case suitable traps have to be provided on the return main, for preserving the steam-pressure within the supply main and radiators. These two systems, in any of their modifications, may also be combined, as is most generally done in any extensive warming apparatus.

In connection with closed circulation, the exceptional character of the single-main distribution calls for some further remarks. The employment of a single main for both supply and return is restricted to buildings where the horizontal distances through which the heat has to be distributed are very short, but where the vertical distances are relatively great. These conditions occur more particularly when the warming is extended to several stories, as in single dwellings or office buildings covering little ground.

There are three principal methods of single-main distribution. In the first, the main is devoid of any internal partition, and is carried up from the boiler at once to the highest point, without taking off from it on the way any distributing branches, all of which are led off from it afterwards in its descending course. Wherever in the ascending course, before the highest point is reached, it has to take an inclined instead of a vertical direction, it must rise at an inclination of not less than 1 in 30 to the horizontal, in order to let the condensed water run back in spite of the current of steam passing in the contrary direction. To ensure the size being large enough, the diameter for a vertical main should be taken at least equal to that furnished by Tables VII. to X. in the Appendix; but for an inclined main (rising 1 in 30) the diameter should be doubled. This first method, however, is much embarrassed by the air which gets entrapped within blind branches that offer no thoroughfare; the radiators are consequently not certain of getting heated, and the apparatus is apt to be noisy for the reasons subsequently explained.

The second method applies to distributions extending through considerable distances horizontally. Here it is more satisfactory to start with a primary circulation through separate inclined supply and return mains, in place of a single main however large or however steep; and to relegate the single-pipe system to a secondary rank, employing it for vertical branches led off from the primary supply main. The course of the primary supply main is a descent with proper slope to the remotest horizontal distance; and, immediately underneath each single-pipe branch rising vertically from it, a receptacle is provided in the primary main, large enough to catch all the condensed water returned from the branch; these receptacles are themselves drained back to the boiler by the return main. From the vertical single-pipe branches, short single offshoots are led off laterally to the radiators in the rooms of the different stories. The air difficulty is avoided with much success by carrying each vertical branch first to the highest point required, free from all lateral offshoots; and supplying the several radiators from its descending or return course, by connections so arranged as to preclude lodgment of air.

In the third method of single-main distribution, which is possibly as satisfactory as either of the two preceding, a single upright pipe is employed, having a hoop-iron partition tightly inserted for separating the ascending from the descending current.

The system of closed circulation requires the boiler to be placed so low as will allow all the return pipes to drain freely back to it above its water-level. This condition has been modified mechanically by the automatic "Albany trap," a device frequently employed for lifting from a lower level part or all of the condensed water, and delivering it into the boiler: it is in fact a displacement pump. The same result has been attained by draining into a closed tank, placed low enough to accommodate all the return pipes, and made strong enough to stand the full boiler pressure with safety; and then employing a steam-pump, either reciprocating or centrifugal, to raise the

water from this tank to the proper level for enabling it to flow back into the boiler, the whole of the circulation being closed from communication with the atmosphere.

Steam mains and branches are apt to be noisy whenever any dipping bend or pocket in the pipes, or any recess in the fittings, allows water to accumulate, and to become cooled below the temperature of the steam supply. In such cases a rapid condensation occurs there, and the steam rushing in carries the water along with it; or sometimes two opposite currents of steam, rushing into the vacuum, meet each other and characteristic noises of rattling and hammering are produced.

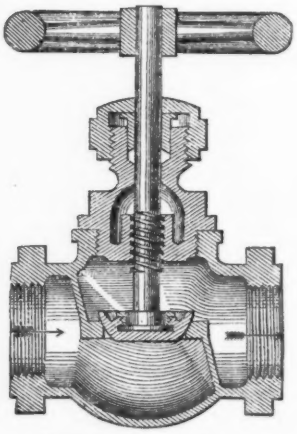
Where separate supply and return pipes are used, whether the system of circulation be open or closed, they should everywhere slope downwards in the direction of the current of steam or of water. A fall of $\frac{1}{2}$ inch in 10 feet (1 in 240) has been found ample to provide against all sag of the pipes or other mechanical defects in the work, and to ensure silent working. When the levels at which the radiators have to be placed do not admit of this slope being continued in the so-called horizontal supply main, vertical breaks are made in the line by the insertion of pipes of larger diameter, which are trapped by check-valves or siphons into the return main. In any extended distribution by separate supply and return mains, the supply main should be connected to the return at the remote ends by the method of open circulation; and between any parallel or not very distant supply and return pipes, occasional drips should be provided, at intervals of say 600 to 1200 times the pipe diameter, siphoned off to prevent any short circuit. Branches upon the horizontal mains, whether supply or return, should be connected upon the top of the main, not at the sides or bottom. Freedom for expansion should be allowed by horse-shoe or S bends, or when practicable by elbows judiciously arranged. Expansion joints are an established fitting for warming apparatus; but their use is not to be approved except in emergency. Repairs are facilitated by substituting at frequent intervals along the mains, in place of some of the screw couplings, cast-iron flange joints, the flanges being screwed upon the tube ends. Main pipes should in all cases be either carried on rollers or suspended, to allow freedom for expansion without straining the joints. The author does not attempt to describe completely all the minutiæ of detail in the construction of the mains for warming by steam; but simply notices some of the appliances commonly employed in America, which have been devised and worked out practically, and are regularly manufactured for general use.

Clothing of Steam Mains.—To prevent loss of heat, steam mains are protected by clothing, as described by Rumford in the last century. Felt is found to perish when applied direct to a surface as hot as 200° Fahrenheit; and Rumford's air-space, formed by enclosing the main within a rather larger casing of thin cast iron, or of sheet iron either plain or tined or zinced, is sufficient for enabling the felt to be employed as the clothing outside the casing. A covering of wire-netting has also been devised. Coats of porous terra cotta or of porous plaster answer, by their low conductivity, to save the felting put outside them. Outside the felt again is applied some suitable sheath or protecting covering that will stand the exposure. When thus clad, the loss incurred in carrying a steam main to any distance—either out of doors, or inside rooms, in passages, in cellars, or in culverts or flues either underground or above—is found to be less than 1 unit of heat for each 100 square feet of external surface of the main itself.

Steam Stop-Valves.—The steam stop-valves, known as "globe valves," are disk or poppet valves, worked by a screw spindle, as shown in the accompanying sketches, Figs. 3 and 4, which represent the "straight-way" make for insertion between two pipes in the same straight line. The annular seating upon which the disk closes is cast in line with the axis of the pipes, and in the middle of the globular or spherical body; a transverse diaphragm above the near end of the seating, and a corresponding wall crossing beneath its far end, close all thoroughfare excepting the aperture in the seating itself. This construction was introduced by the author in 1849, and was immediately followed by all makers. The same globular form for the body has been adopted also, as a matter of symmetrical appearance, for the three makes

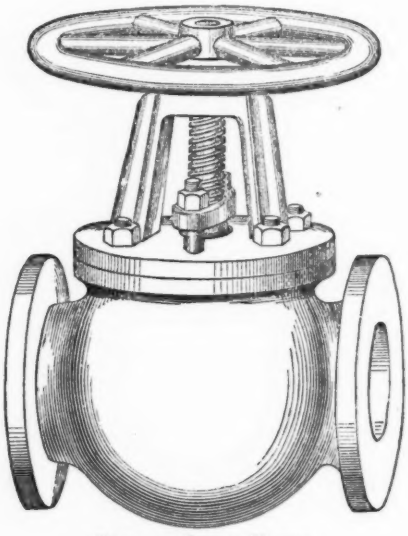
of valve employed to unite pipes at right angles: the first, known as angle valves, unite two pipes, of which one is in line with the valve-spindle, and the other is at right angles to it; the second, called corner valves, unite two pipes at right angles to each other and both of them at right angles to the spindle; and in the third, or cross valves, two pipes in line with each other and at right angles to the valve spindle are united to a third pipe in line with the spindle. The principal dimensions of straight-way globe-valves are given in Table IV. for the various sizes in use. In general the smaller valves, not exceeding $1\frac{1}{4}$ inch in diameter of opening, are wholly of gun-metal; the larger are commonly and preferably made with cast-iron bodies and gun-metal fittings. The smallest valves, from $\frac{1}{4}$ inch up to $\frac{1}{2}$ inch inclusive, have the disk solid with the spindle, and have an ordinary stuffing-box with external gland. Valves of $\frac{3}{4}$ inch and upwards have the disk loose from the spindle, and the spindle is made with a cheese-head let into a recess in the disk, and secured there by an annular nut; they have also a separate internal gland, tightened by a screwed cap, as shown in Fig. 3. The valve-spindles are screwed left-handed, with a double thread of square section; up to 3-inch valves the spindles are screwed to work inside the casing, as in Fig. 3; above that size the screwed portion is outside the casing, and works through a nut in a stool bolted on the casing, as in Fig. 4. In all the valves the disks and seatings have their surfaces of contact shaped spherical; and the disks are without wings to guide them. Above the 3-inch size the nozzles of the cast-iron bodies are generally flanged instead of tapped. This construction is particularly satisfactory for large valves, 10-inch and 12-inch; the last is the largest size usually kept in stock. The resistance presented by a globe-valve to a current flowing through it is assumed at half as much again as the resistance of a sharp right-angled elbow.

FIG. 3.



TAPPED GLOBE VALVE.

FIG. 4.



FLANGED GLOBE VALVE.

Radiators for diffusing heat.—In respect to the radiating surfaces for the diffusion of the heat, there are three distinct classes of warming apparatus in use in America, according to the object to be effected.

First, there is the apparatus for warm-

TABLE IV.—DIMENSIONS OF STRAIGHT-WAY "GLOBE" VALVES.

Size.—Diameter of opening in seating.	Body.—Gun-metal or Cast-iron.	Nozzles.—Tapped or Flanged.	Length over all.	Diameter of Flanges.	No. of Bolts in each flange.
Inches.			Inches.	Ins.	No.
¼	Gun-metal	Tapped	1.45		
⅜	do.	do.	1.78		
½	do.	do.	2.20		
¾	do.	do.	2.65		
1	do.	do.	3.30		
1¼	do.	do.	3.85		
1½	Gun-metal	do.	4.35		
	Cast-iron	do.	5.10		
2	Gun-metal	do.	5.30		
	Cast-iron	do.	5.90		
	Cast-iron	Flanged	5.75	6	4
2½	Gun-metal	Tapped	6.75		
	Cast-iron	do.	7.30		
	Cast-iron	Flanged	7.25	7	4
3	Gun-metal	Tapped	7.75		
	Cast-iron	do.	9.25		
	Cast-iron	Flanged	9.25	7½	4
3½	Cast-iron	Tapped	10.25		
	do.	Flanged	10.25	8	5
4	do.	Flanged	11.25	9	5
5	do.	do.	13.25	10	6
6	do.	do.	15.25	11	6
8	do.	do.	19.00	13½	8
10	do.	do.	23.00	16	10
12	do.	do.	27.00	19	10

ing rooms by so-called direct radiation; that is, by means of radiating surfaces exposed in the rooms themselves.

Secondly, there is the apparatus for what is sometimes called indirect warming, by means of currents of air: the heated surfaces are placed in a chamber, through which a limited supply of air is allowed to pass on its way into the room. In neither of these two methods is the warming accompanied by any systematic ventilation.

Thirdly, there is the apparatus for both warming and ventilating, arranged so that the warming shall take effect upon the whole supply of air admitted for ventilation.

The first and second methods are, with rare exception, employed for all dwellings, offices, and mills; the third is reserved for hospitals, asylums, public buildings, and the like. On the first and second plans, the warming apparatus is required to be capable of maintaining a comfortable warmth in the very coldest weather, when the outdoor temperature ranges as low as 15° below zero Fahrenheit; whence it is argued in favor of these two methods that in moderately cold weather their warming capacity will be so largely in excess as to admit of ample ventilation by opening windows or doors at pleasure. There are some who condemn altogether the first method or direct radiation, and strongly recommend the second or indirect, on the ground of the supply of fresh air secured by the warming currents in the latter method. But, as a rule, this supply of fresh air is inadequate; and, with the prevalent construction of apparatus, the temperature of the warming currents on issuing from the heating chambers is too high, and cannot be controlled; and the regulation of the warmth in the room has to be effected by opening wider or partially closing the hot-air inlet from the heating chamber into the room, thereby altering the admission of fresh air. On the score of ventilation, the second method is perhaps preferable to the first, because for any temperature of the external air the second method does certainly supply some admission of fresh air, however varying and inadequate. But in the estimation of the community at large, the first method—direct radiation from surfaces exposed in the room itself—offers the pre-eminent advantage that the warming of a room is effected with great certainty and rapidity, and is under the immediate control of the occupant. Direct radiation has also the merit of requiring the smallest consumption of fuel; and although the cost of the apparatus is increased by the superior finish required for the radiators, which are here exposed to view, and by the greater length and complication of the mains, in comparison with the indirect method, yet the increase is about counterbalanced by the saving in extent of radiating surface and in diameter of mains with the direct system; and the special constructions of heating chambers, flues, and dampers, attending the indirect system, make the latter the more costly mode of warming any building.

The conditions of American ventilation and warming, in relation to health and

comfort, differ materially from those prevailing for the same latitudes in Great Britain and other western parts of Europe. The temperature agreeable to Americans in cold weather is about 70° Fahrenheit on the Atlantic coast, rising to 80° or 85° for inland localities and for the severest and driest cold. The American and European requirements differ therefore so completely, that in the tables given in the appendix no attempt has been made to establish any relation between the area of radiating surface in the warming apparatus and the extent of building to be warmed, whether as regards floor area, outside surface of walls or roof, and cubic capacity, or in connection with ventilation for varying numbers of persons or of lights; the tables are presumed to be equally applicable to data derived from experience on either side of the Atlantic.

In the buildings warmed in America by direct radiation there is generally no provision whatever of air-flues, either inlet or outlet, to aid ventilation. This is true for dwelling-rooms, offices, and mill or factory rooms. Recently in many of the better living-rooms, and in offices also, open fire-places have been built, mainly for show, but possessing some utility in producing a little radiating heat for speedy warming. Fire-places, while valuable for ventilation, are chiefly important from the advantages they offer for the attainment of an equable temperature throughout a room. The outlet aperture in any room must be at or near the floor, at any rate in cold weather, so as to remove the air from the bottom of the room where it is coolest. Although the fouler air is of course at the top of every warm room, yet the discomfort consequent upon removing the warmer air from the top, and allowing the cooler air to stagnate below, is found too serious to be endured. A healthy atmosphere would indeed be maintained throughout the room, provided that, in removing air enough from the top, care were taken to supply an equal quantity of fresh air by a diffused admission suitably arranged for comfort; but complete ventilation will always fail to receive due consideration wherever it clashes with comfortable warmth. Gas lights are used, to the exclusion of all others, in American dwellings; and no inconvenience or unhealthiness is considered to result from the unventilated burners. It is fully recognized that the vitiation of air by the combustion of gas is not organic; and although, when the products of combustion are in excess, the air becomes unbreathable, it is by no means the source of disease.

In warming a room by direct radiation, the proper situation of the radiating surface for the attainment of an equable temperature is in some respects a moot question. As long ago as 1846 Dr. Morrell Wyman* called attention to the way in which heat was distributed in any room, when the source of heat was situated as usual at the back of the room, against an inner wall, with windows and exposed outside wall at the front or side of the room; and he showed that a layer of highly heated air collected next the ceiling, whence descended a sheet of air along the cooler wall and window surfaces, becoming itself cooled in its descent, until at about the height of a man a uniform temperature of comfortable warmth was established throughout the room. It is in conformity with this principle that, in the practice of warming in America by means of a hot-air furnace, the hot-air flues from the furnace are generally led up inside an inner wall, remote from windows or outside walls, and a tolerably equable and comfortable warmth is obtained by admitting into the room a limited quantity of very hot air, sometimes nearly as hot as 400° at the inlet. A closed stove, situated likewise against an inner wall, proves similarly effective; but with an open fire the draught of cold air, which from some source or other takes its course along the floor to the fire-place, seriously impairs the desirable uniformity of temperature in a room.

In warming by steam with direct radiating surfaces, the practice for many years was to arrange the steam pipes in lines or groups, called coils, along the bottom of the outside walls or under the windows. But present usage seems to indicate that better results can be obtained by placing the radiating surface where it

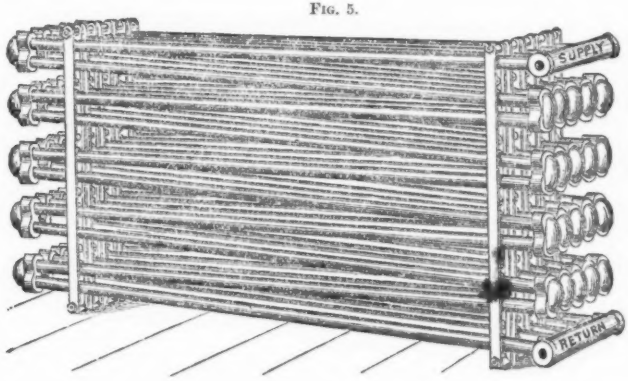
* Dr. Wyman published in Boston in 1846 "A Practical Treatise on Ventilation and Warming," containing much of novelty at the time and of lasting excellence. The work, now out of print, is as suggestive as Walker's "Hints on Ventilation," with which it may be compared.

will apparently be warming what is already the hottest part of the room; in reality it then promotes the natural circulation, whereby the proper diffusion of the warmth is aided, instead of being retarded. For warming rooms in mills, the most recent practice is to place the direct radiating pipes in rows overhead, suspended a foot or two from the ceiling, and two or three feet from the outside walls. Although by this arrangement the heat would apparently be expended in the top of the room, yet very satisfactory results are thereby obtained, in regard both to equability of warming and to efficiency of radiating surface.

to branch tees or heads. In a few exceptional cases, radiators of peculiar shapes are specially constructed. In all cases the coils must have either vertical or horizontal elbows of moderate length, for allowing each pipe to expand separately and freely. Sometimes short lengths of pipe are coupled by return-bends, doubling backwards and forwards in several replications one above another, and forming what are called "return-bend coils;" and when several of these sections are connected by branch tees into a compact mass of tubing, the whole is known as a "box-coil," Fig. 5.

As the amount of heat given off from

Fig. 5.



Return-Bend on Box-Coil.

In rooms warmed by direct radiation, the attempt has repeatedly been made to effect the ventilation by admitting air direct upon the radiating surfaces; which are then placed under the windows, so as to intercept any descending currents of cooled air. Not much success has attended this plan, inasmuch as through some of the inlet apertures outward currents of warm air are then apt to escape from the room, producing no perceptible warming of the external air; while through others a flood of cold air enters from the outside, and, pssing only some one coil or radiator, gets hardly warmed at all, the effect being far from comfortable.

Construction of Radiators.—For warming by direct radiation, the radiators usually consist of coils, composed of ¾-inch and 1-inch steam-pipes, which are arranged in parallel lines and are coupled

the radiator cannot be satisfactorily controlled by throttling the steam supply, it is usual to divide all radiators into sections, each of which can be shut off from the supply and return mains, separtely from the rest of the sections. This method of regulation applies to radiators for indirect heating as well as for direct.

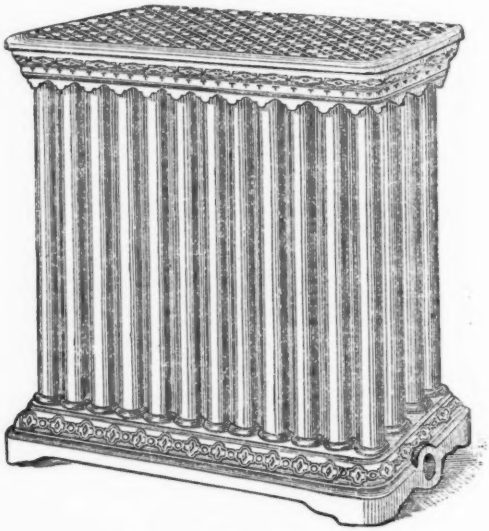
Vertical-pipe coils, Fig. 6, constitutes a distinctive form of radiator now largely used. In these a number of short upright 1-inch tubes, from 2 feet 8 inches to 2 feet 10 inches long, are screwed into a hollow cast-iron base or box; and are either connected together in pairs by return-bends at their upper ends, or else each tube stands singly with its upper end closed, and having a hoop iron partition extending up inside it from the bottom to nearly the top. The supply of steam is admitted into the bottom casting; and the steam

on entering, being lighter than air, ascends through one leg of each siphon pipe and descends through the other, while the condensed water trickles down either leg, and with it the displaced air sinks also into the bottom box. For getting rid of the air, a trap is provided, having an outlet controlled by metallic rods; as soon as all the air has escaped and the rods become heated by the presence of unmixed steam, their expansion closes the outlet.

rosion, although pipes rusted off from the outside are by no means uncommon.

For indirect radiating surfaces, the box coils are the forms most used. The chambers or casings for containing them are made either of brickwork, or often of galvanized sheet-iron of No. 26 gauge, with folded joints. The coils are suspended freely within the chambers, which are themselves attached to the walls containing the air inlet flues. Besides coils of wrought-iron tubes, cast-iron tablets

FIG. 6.



VERTICAL-TUBE RADIATOR.

One construction of direct radiating surfaces that has had repeated trials consists of tablets of thin sheet-iron. These were proposed and used by James Watt, and were stayed by having their sides indented with recesses and quilted together. The cause of failure lies in the presence of air inside an iron vessel in which steam is condensing. Under no other condition of exposure does iron perish so rapidly. In the ordinary closed circulation, the wrought-iron tubes and cast-iron fittings are practically imperishable internally, owing to the entire exclusion of air. At any rate, during more than thirty years' experience the author has never seen a pipe destroyed by rust by internal cor-

or hollow slabs, having vertical surfaces with projecting studs or ribs, have been extensively used for the radiating surfaces in warming houses with low pressure steam. They are like the tablets common in England for warming with hot water. Their great advantage has been found to lie in their small height; as little as 6 to 8 inches is height enough to warm sufficiently the current of fresh air traveling only that distance in contact with their heating surface.

Ventilation combined with Warming.—Where systematic ventilation is carried out in conjunction with warming, the indirect radiators and chambers just described are employed. The regulation of

the warmth to be supplied by the apparatus can be effected by dividing the coil into independent sections, one or more of which can be shut off at pleasure, as previously mentioned. But in combination with systematic ventilation the warming can also be more effectually controlled by so arranging the casing or chamber containing the coil, that the whole or any part of the fresh air entering can be made to pass through the coil and be warmed, or to "by-pass" the coil and escape warming; the warmed and unwarmed currents are then mingled in a mixing chamber or flue, whence a supply of fresh air suitably tempered flows into the room. This method renders it feasible for the occupant of the room to control the tempering of the air, and thereby to regulate the warming of the room; it also ensures the constant supply of a definite quantity of air.

Where a blowing fan is employed as a means of impelling a current of air through a building, a further improvement for large and extensive warming apparatus is proposed, which consists in placing a large auxiliary warming coil at the entrance of the main supply flue that leads from the fan into the building.* Provision is made for using either the whole of the auxiliary coil or only a portion of it, and it is also provided with by-passages and regulation shutters or dampers, so that the air entering the main supply-flue to the building can at all times be warmed to a uniform temperature of say 50° Fahrenheit. The air thus warmed can be allowed to pass along flues wheresoever situated--underground, through cellars, or under passages of a building—without much loss of warmth, and without danger of injuring foundations by freezing. From these flues the air is admitted into the coil chambers previously described, where it undergoes the further warming requisite to give the desired temperature in the rooms. By this means, while the volume of fresh air entering a room remains constant, its warmth may be regulated to any temperature from 50° as a minimum up to 120° as a maximum. The extent of radiating surface distributed inside the rooms will by this arrangement be only about one-half of the total that has usually to be provided where no auxiliary warming coil is employed; while the large auxiliary coil itself has only about 40 per cent. of the total surface usually provided in its absence. Hence this arrangement actually saves about 10 per cent. of warming surface, irrespective of the saving in cost of steam mains when the boilers are placed near the fan or entrance to the main supply-flue.

The blowing fan generally employed in America for ventilating large buildings is that described by the author in a former paper to this Institution. The exhaust steam from the engine driving the fan is utilized for warming some large coil; and any deficiency in quantity is made up by a supply of live steam taken direct from the boiler through a "differential pressure" valve, which is of common use where exhaust steam is employed for warming.

Examples of extensive Warming by Steam.—As an example of warming on an extensive scale may be taken a large office in New York, of which the following are the particulars:—

Total number of rooms, including halls and vaults	286
Total area of floor surface....sq. ft.	137,370
Total volume of rooms.....cub. ft.	1,923,590
Number of constant occupants during office hours	650
Maximum average of occupants at any time	1,300
Volume per occupant, excluding vaults.....cub. ft.	1,443

In addition to the steam for warming, the boilers all furnish steam for the engine power expended in working lifts or elevators in the building, in pumping the water supply, and in electric lighting; and the same boilers also furnish steam for motive power and for warming to other buildings at several hundred feet distance, this extra service absorbing about one-third of the total boiler capacity. The boilers are eight in number, and have altogether 173 square feet of grate area, with about 8,000 square feet of heating surface. The lifts or elevators convey about two million persons per year.

A second example is furnished by the State Lunatic Asylum at Indianapolis:—

* This method is believed to have been first proposed by Major-General M. C. Meigs, U.S.A., when in charge of erecting the U.S. Capitol at Washington, in 1856.

Length of frontage of building, more than		2,000 lin. ft.
Total volume of rooms		2,574,084 cub. ft.
Warming apparatus	Indirect radiating surface	23,296
	Direct	10,804
	Total	34,100 sq. ft.
Boilers	Grate area	180 sq. ft.
	Heating surface	5,863 sq. ft.

This warming apparatus was constructed by the Walworth Manufacturing Co. of Boston, after the plans of their engineer, Mr. L. R. Greene, contrary to whose advice, however, the ventilation is effected by chimneys instead of by a blowing fan.

As examples of the extent of heat transmission from a central source may be mentioned the hospital at Columbus, Ohio, and that at Buffalo, New York, each warmed by steam, the former having a linear frontage of 2,280 feet, and the latter of about 3,000 feet.

While these dimensions give a notion of the magnitude of the warming apparatus for numerous large buildings, they fail to convey any idea of the very general prevalence of warming by steam in any of the commercial cities of America. The boilers in use for the purpose at any one warehouse are made to supply steam, for warming and for motive power, to any distance and to any extent within the limit of their capacity. The system is adopted in Boston and New York for the larger residences in flats or stories, which are now rapidly coming into favor. There appears indeed no limit to the future extension of systematic steam supply for warming and for motive power; and every facility is afforded for its growth, from the fact of the necessary mechanical details having already been fully worked out.

RECENT ARMOR-PLATE EXPERIMENTS.

From "Nature."

At the conclusion of their labors the "Iron Plate Committee" reported, in 1865, that the best material for the armor of war-ships was wrought iron of the softest and toughest nature. Steel, or steely iron, or combinations of iron and steel, were all pronounced unsuitable for the purpose, after a long course of careful experiments. Accepting this verdict, the designers of armored ships continued to specify for soft iron armor, the makers of guns and projectiles aimed at the perforation of this kind of armor, and the manufacturers sought to secure the desired qualities of softness and toughness in the thicker and heavier plates which they were constantly being called upon to produce. All the armored ships built from 1860 to 1876 were "ironclads," and in that time the thicknesses of armor plates carried on the sides or batteries of completed ships had advanced from 4½ inches to 14 inches, while the weights had risen from 4 or 5 tons to 20 or 25 tons. Greater aggregate thicknesses of iron had been arranged for prior to 1876. For example, the Inflexible had been designed to carry 24 inches of iron on her sides, but this was in two layers of 12-inch plates. The adoption of the so-called "sandwiched fashion" of armor plating was based upon experiments made at Shoeburyness, and it had certain advantages of a constructive character; it also enabled broader and longer plates to be produced within the fixed limits of weights with which the manufacturers could deal, and enabled them to insure excellence of quality which might not have been so certain of attainment in plates of 20 inches or upwards in thickness.

While the two great Sheffield firms and their rivals in France were thus developing the manufacture of iron armor plating, the Creusot Company, of which M. Schneider was the head, were attempting to reverse the verdict against steel armor, and to produce specimens which could hold their own against the best iron armor of equal thickness. The Italian Admiralty brought the claims of the rival materials to the test of experiment at Spezia in October, 1876. In order to decide on the kind of armor to be used on the Duilio and Dandolo, specimen targets were erected and a series of firing trials made against them on a scale of unprecedented magnitude. A gun weighing 100

tons, manufactured at Elswick, was brought to bear upon targets protected by iron or steel plates 22 inches thick, backed by great masses of timber and strong supports. Other guns of considerable weight and power were also used, but their performances were overshadowed by those of the monster weapon. The results of these trials may be briefly summarized. Against the 10-inch and 11-inch guns the 22-inch iron plates had a decided advantage over the steel plate of equal thickness. The penetration was somewhat greater in the iron plates, but the steel plate cracked badly. On the other hand, when the 100-ton gun was brought against the targets, the iron plates and their backings were completely perforated as well as broken up; whereas the steel plate, although smashed to pieces, prevented the shot from passing through the backing. Various opinions were formed as to the deductions which should be made from these trials. On the one side it was urged that as steel plates of great thickness could be gradually cracked and destroyed by guns incapable of perforating them, steel ought not to be used instead of iron, which could be battered by a great number of projectiles from such guns, and be neither perforated nor cracked. On the other side, it was maintained that there was small probability of any single armor plate on a ship's side being struck repeatedly in action, and consequently that the material should be preferred which could best resist perforation by a single projectile from the most powerful gun, even if the resistance to perforation involved the partial destruction of the plate struck. The Italian authorities adopted the latter view, and the Duilio and Dandolo have steel armor, being the first ships protested in that manner.

Although these steel armor plates were made in France, the French authorities did not follow the Italian lead and abandon iron armor. Nor was a similar course followed in England. Change was seen to be inevitable, but it was endeavored to make the change in a direction which should combine the high resistance to perforation of steel with the power to resist cracking and disintegration possessed by tough rolled iron. To Messrs. Cammell & Co., of Sheffield, belongs the honor of taking the lead in this direction; Messrs. Brown speedily followed, and the Admiralty gave substantial assistance in the conduct of the necessary experiments. In the earlier stages many failures and disappointments were experienced, but eventually better results were obtained, and "steel-faced armor" became recognized as the substitute for iron on English war-ships. Steel-faced armor, as the name implies, consists of a rolled iron back-plate, on the face of which is welded a layer of steel. The hard steel face resists perforation, and breaks up or deforms the projectiles, while the intimate union of the tough iron back with the hard steel face prevents the serious cracking which occurs in steel alone. Curiously enough, the idea was not merely an old one, but a small plate made on this principle, 4½ inches thick, had been fired at in 1863. This early steel-faced plate was broken into two pieces at the first shot of a light gun, and was condemned by the Iron Plate Committee. Fourteen years later plates of a similar character, so far as the combination of steel and iron is concerned, but of improved manufacture, were successfully resisting three shots, either of which would have perforated an iron plate of equal thickness.

The first steel-faced plates were used on the Inflexible's turrets; they were 9 inches thick, worked "sandwich-fashion" outside 7-inch iron armor. It was part of the contract that a test-piece from each steel-faced plate should be fired at with a 12-ton gun, and should receive three shots without being broken up or perforated. This was considered to be a very severe test at the time, and undoubtedly was so when the novel conditions of the manufacture are considered. It was successfully met, however, and from that time onwards the manufacture has steadily improved. As an indication of what has been done, it may be stated that steel faced plates, 11 inches thick, have received no less than eight shots from the 12-ton and 18-ton muzzle-loading guns, with battering charges and at 10 yards' range, without perforation or very serious cracking, this enormous "punishment" having been sustained by an area of 48 square feet only. Most of the trials made against steel-faced armor have been against plates from 10 to 12 inches in thickness. For thicknesses up to 12 inches it is

probably within the truth to say that for *normal impact* the steel-faced plates of recent manufacture have been equal in their resistance to perforation to iron plates 25 to 30 per cent. thicker and heavier. For oblique impact the hard armor is probably still more superior to iron, glancing the projectiles at angles of obliquity when they would have "bitten" into the iron. A few experiments have been made in this country and abroad on much thicker steel-faced plates, ranging up to 18 or 19 inches in thickness, and of these the most recent and important are the trials made at Spezia in November, 1882. Three targets were constructed for these trials, the armor plate on each being nearly 11 feet long, 8½ feet wide, and 19 inches thick. One of the targets was covered by a steel-faced plate made by Messrs. Cammell, another by a steel-faced plate made by Messrs. Brown and, the third by a steel plate made at Creusot. All three plates were similarly backed and supported by 4 feet of oak; the Creusot plate was fastened by no less than 20 bolts, and the Sheffield plates had only 6 bolts each. Against these targets the 100-ton muzzle-loading gun was brought into action. At first the powder charge used (329 lbs.) was that which gave such a velocity to the chilled cast-iron projectiles—2,000 lbs. in weight—as would have perforated a 19-inch iron armor plate. The actual penetrations were from 3¼ to 5 inches in the steel-faced plates, and 8¼ inches in the steel plate, showing that the actual superiority of all the plates over iron considerably exceeded the estimate. The steel plate did not crack at the first shot: the steel-faced plates did, but not to any serious extent. Next followed a more severe attack, the powder charge being increased to 480 lbs., giving the projectiles a velocity estimated to be capable of perforating about 24 inches of iron armor. The total energy of the projectile moving at this velocity exceeded 33,000 foot-tons. All the plates were broken into pieces by this terrific blow. The steel plate was split into six pieces, but the numerous bolts held these pieces in position, and still preserved the defensive power of the target. Each of the steel-faced plates was broken into five pieces, and on account of the fewness of the bolts these pieces fell to the ground, leaving the targets uncovered. The whole of the chilled cast-iron shots were broken up on impact, and the penetration into the steel-faced plates was less than that in the steel plate. At this stage the comparative tests ended. A third round was fired, with the heavier charge and a steel projectile against the steel plate. The shot was stopped, the penetration was only 7 inches, but the plate was broken up, and the backing seriously splintered. A fourth round was fired at this target, and completely wrecked it.

On a review of all the circumstances of the experiments, it must be admitted that the greatest success was attained by the steel plate, although this must be attributed rather to the number and excellence of its fastenings than to superiority in quality of the plate over the steel-faced plates. The latter proved themselves less penetrable than the steel plate, and had rather the advantage as regards fracture at the end of the first two series of rounds; but they were insufficiently secured. One definite lesson to be learned from these experiments is, therefore, that a larger number of bolts is needed for a given area of steel or steel-faced armor than has been commonly used. Another lesson taught by these trials is that the steel armor plates of Creusot manufacture in 1882 are far superior to those made six years earlier. It is not at all probable that light guns such as broke the 22-inch steel plate to pieces in 1876 would have been equally effective against the 19-inch plate recently tested. In both cases the plates were made specially for the firing tests, and they may not have been "merchantable articles" in the sense of representing large quantities of steel armor. But nevertheless this 19-inch plate shows what can be done with steel, if cost is of secondary importance. Authoritative statements are wanting of the actual processes of manufacture, or of the cost of production. It is reported that the 19-inch plate was hammered down from an ingot three or four times as thick as the finished plate, and that the face was oil tempered. If this is correct, the cost must be high, and probably as great as, if not greater than, that of steel-faced plates. Moreover, if such an amount of "work" has to be put into steel plates in their conversion from ingots into the finished forms, then no

great economy or advantage can result from the power which the maker has to cast steel ingots in special shapes or sectional forms. The Creusot Company use a soft steel containing perhaps three-tenths to four-tenths per cent. of carbon, give it toughness by means of a large amount of hammering, and harden the face by oil tempering. On the contrary, the Sheffield firms, as the result of numerous experiments, use a hard steel for the face, the percentage of carbon amounting to about twice that in the Creusot plate, and support this by a tough iron back. With this hard steel oil tempering does not appear to be beneficial, although with softer steel it undoubtedly is an advantage. These steel-faced plates which were tested at Spezia were really samples of large quantities made at Sheffield in the same manner. Probably equally good results would have been obtained if any one of the batch of plates represented had been selected for test. In this respect, therefore, there is a marked difference between the test to which the two manufactures were subjected.

As between the steel and steel-faced plates tried at Spezia, we may assume that there is no notable difference in resistance to perforation or to fracture. Possibly, with equally good and equally numerous fastenings, the steel-faced plates would have had some slight advantage, and in other trials mentioned later on steel-faced plates have had a decided advantage. Supposing no important difference to exist, then the choice between the two kinds of armor will be governed by their relative prices; and how these compare we have no means of judging, but it seems probable that the steel-faced plates would be at least as cheap as steel plates made in the manner described above for the steel test-plate.

It may be convenient in this connection to briefly describe the mode of manufacture of steel-faced plates. Messrs. Cammell prefer to pour the molten steel on to the face of a wrought-iron plate which has been brought to a good welding heat. The layer of molten steel is surrounded by a frame of wrought-iron which has previously been attached to the iron plate; and it is pressed against the surface of the iron plate by a cover carried by an hydraulic ram, until the welding is complete and the steel has solidified. Messrs. Brown prefer first to roll a steel face-plate, as well as an iron back-plate, and then to raise both to a welding heat; the molten steel is afterwards poured into a space left between the two, and hydraulic pressure is applied until the solidification has taken place. The remaining processes are similar in the practice of both firms. After welding has been completed the whole mass is reheated and rolled down to the finished thickness of the armor plate. The steel face is usually about one-half the thickness of the iron back, and it is a curious fact that the iron and steel maintain their relative thicknesses as the rolling proceeds, even when the reduction in thickness during rolling is very considerable. This reduction varies from one-half for thin armor-plates, up to 10 or 11 inches in finished thickness, to one-third with 18 to 20 inches of finished thickness. Some competent authorities consider that too little work is done in the rolls on the thicker plates, but there is a need for further experiment to show whether this view is correct. Whatever may be the cause, it would seem that the best results so far have been obtained with steel faced plates below 12 inches in thickness.

Simultaneously with the Spezia experiments another competition was proceeding, near St. Petersburg, between steel faced and steel armor. The plates tested were 12 inches thick, 8 feet long, and 7 feet wide. They were first fired at with the 11-inch breech-loading gun, throwing a 550-lb. chilled cast-iron prejectile, with a powder charge of 132 lbs. The velocity of the shot was 1,500 feet per second. Messrs. Schneider supplied the steel plate, which was fastened with twelve bolts. Messrs. Cammell made the steel-faced plate, which had only four bolts in it. The first blow on the steel plate broke it into five pieces; the projectile was destroyed, but it penetrated 13 inches into the target. A blow of equal energy on the steel-faced plate produced only a few unimportant cracks in the steel, and the penetration was about 5 inches only. Three out of the four bolts were, however, broken. A second shot was then fired at each plate with 81 lbs. charge. The steel plate was broken into nine pieces, and the penetration was 16 inches; whereas, on the steel-faced plate the principal effect produced was to break

the only remaining bolt and to let the plate fall to the ground, face downward. The back of this plate was perfect, and the target behind the plate was uninjured. In this trial the steel-faced plate proved greatly superior to the steel, but had insufficient fastenings. It is proposed to increase the bolts in number, re-erect the plate, and continue the trial, of which the further results cannot fail to be interesting.

This contest between steel and steel-faced armor must not be allowed to withdraw attention from the great superiority of both, in certain respects, to iron armor. Even as matters stand, either of these modern defences is greatly to be preferred to their predecessor. Against this hard armor chilled cast-iron projectiles break up in a manner never seen with soft iron. Projectiles of this kind are virtually impotent, and must be replaced by more expensive, harder projectiles, if steel or steel-faced armor is to be attacked. Even with steel projectiles results cannot be obtained such as were possible with iron armor. Perforation of armor by shells carrying relatively large bursting charges is no longer a possibility; and the heaviest gun yet made cannot drive its projectiles through a thickness of hard armor only three-fourths as great as the thickness of iron which it could perforate.

The use of steel and steel-faced armor will involve many experiments to determine not merely what descriptions of projectiles are best adapted to damage or penetrate it, but what are laws of the re-resistence of such armor to the penetration and disintegration. All the formulæ based on experiments with soft iron armor and chilled cast-iron projectiles are inapplicable under the new conditions. Perforation is no longer to be feared as the most serious damage likely to happen to armor plates; more moderate thicknesses of hard armor suffice to stop the projectiles from the heaviest guns than would have been considered possible a short time ago. Instead of perforating 19 inches of steel or steel-faced armor, the projectile of the 100-ton gun with a given velocity only penetrates 8 inches into the plates. But, on the other hand, the possible disintegration and fracture of the armor plates are becoming important matters. Makers of armor plates have to endeavor to produce materials which shall resist fracture as well as penetration, and the only proof of their success or failure is to be found in the results of actual trials. Experiments are equally essential to progress in the manufacture of guns and projectiles. The example set by Italy must be followed; the necessary experiments must be on a large and costly scale, and they may lead to many departures from former practice. But if real progress is to be made in the armor and armament of ships, it must be prefaced by experiments beside which those of the former Iron Plate Committee will appear insignificant.

In conclusion, it may be stated that although iron armor has been practically superseded for the sides and batteries of war ships, it is still preferred for decks. Experiments have shown that for angles of incidence below twenty degrees, and for such thicknesses—not exceeding 3 or 4 inches—as are used on decks, good wrought-iron is superior to both steel and steel-faced plating. The explanation of this departure from the laws which hold good for thicker plates and greater angles of incidence cannot be given here, but the fact has been established by elaborate trials made in this country and abroad.

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—Regular meeting April 18th, 1883.

The Secretary stated that arrangements were well advanced for the Convention of the Society to be held at St. Paul and Minneapolis, beginning June 20th. The ordinary meetings to be held at St. Paul, one meeting at which the President's address will be delivered to be held at Minneapolis.

A paper by the late Wm. R. Morley, M. Am. Soc. C. E., was read by the Secretary upon the subject of the "Proper Compensation for Railroad Curves upon Grades." Mr. Morley expresses he opinion that the resistance due to curvature is measured not by the length of radius but by the length of train or what is the same, by the ruling grade and that while there may be some increased resistance due to radius, that may be largely overcome by the elevation of the outer rail, and that in the location of a railroad the length of train or ruling grade should be made the basis of compensation, and not the radius of curvature.

He gives examples in his experience where the practice of compensation with reference to the radius resulted upon steep grades in a decided excess of compensation, and in a noticeable increase of speed of the train upon curves.

He also gives the rules adopted by him in his practice which were as follows:

Rate of max. grade.	Per degree compensation.
.00 to .70 per 100 feet.	.06 per 100 feet.
.70 to 1.60 "	.05 "
1.60 to 3.00 "	.04 "

The paper was discussed by Messrs. Bogart, Chanute, T. C. Clarke, Emery, Forney, MacDonald, North, Wm. H. Paine, D. Ward, and L. B. Ward.

In the discussion Mr. Chanute referred particularly to a paper by S. Whinery, M. Am. Soc. C. E., published in the Transactions of the Society in 1878, on the "Resistance of Curves," and the discussion upon that paper, stating that the theoretical resistances determined by Mr. Whinery agreed very closely with the practical results obtained by experiments upon ordinary wheels at low speed, and that the result was an addition of about one-half pound per ton, per degree, to the resistance on straight lines, and that the equation for curvature resulting from this was about half of what Mr. Morley has adopted for his lighter grades.

The paper will be published in an early number of the Transactions, and will be discussed with others at the approaching convention.

Meeting of May 2d, 1883.

The death of Milton Courtright, Fellow of the Society, was announced. Photographs of a dredge and snag boat and of a drilling scow of peculiar construction used by J. H. Striedinger, M. Am. Soc. C. E., in the improvement of the Magdalena River, South America, were presented. The following candidates were elected: As members, George H. Benyenberg, Milwaukee, Wis.; J. W. Bishop, St. Paul, Minn.; T. L. Griswold, Lagos, Mexico; Randell Hunt, Fargo, Dakota; F. W. Kimball, Milwaukee, Wis.; C. J. Poetsch, Milwaukee, Wis.; A. H. Scott, Milwaukee, Wis.; D. C. Shepard, St. Paul, Minn.; J. A. Smith, Indianapolis, Ind.; G. H. White, Minneapolis, Minn.; as Junior, G. A. Lederle, Bismark, Dakota.

A paper by Professor J. A. Waddell, M. Am. Soc. C. E., "Suggestions as to the Conditions Proper to be Required in Highway Bridge Construction," was read by the Secretary and discussed by Messrs. Cooper, Haight, and MacDonald.

ENGINEERS' CLUB OF PHILADELPHIA. Meeting, April 7th, 1883.

Mr. Arthur Winslow, presented, and described the derivation of, Tables for Stadia Reductions, which furnish expressions for horizontal distances and differences of elevation corresponding to 100 foot stadia readings for 2° up to 30°, on the supposition that the rod be held vertically, and the stadia wires be equidistant from the center wire. They are not mere reductions of inclined distances to their horizontal and vertical exponents, but embody certain corrections necessary from the facts: 1st, that, with horizontal sights, the length cut off by the stadia wires on the rod is not directly proportional to its distance from the center of the instrument but from a point, at a distance in front of the object glass, equal to its principal focal length; and, 2d, that, with inclined sights, a correction has to be made for the oblique view of the rod. Both the distances and elevations in these tables are given in feet; they are adapted to use with a telescope whose object glass has any focal length, and with a rod which is so graduated that the spaces cut off on it by the stadia wires are directly proportional to its distance from a point at a distance in front of the object glass equal to its principal focal length, differing in these respects from the tables issued by the Engineer Department, U. S. A.

Mr. C. O. Hering presented a table giving the number of electrical units of work and power corresponding to all mechanical units, such as horse powers, foot pounds, kilogrammeters, heat units, etc., and the number of the latter corresponding to each of the former, illustrating the same with some examples.

The Secretary presented, for Mr. P. F. Brendlinger, a description of the Bridge Specifications, of the P. McK. & Y. R. R., with specimen copies of the same.

Mr. C. O. Hering exhibited specimen of cable for electric wires.

ENGINEERING NOTES.

BOSTON WATER-WORKS.—An elaborate description of the additional supply of water for the city of Boston from Sudbury River, compiled by Mr. A. Fteley, the resident-engineer upon the work during its construction, has just been issued by the city government in a large, finely printed, and copiously illustrated, volume. The works for supplying Boston with water from Sudbury River consist of three storage-reservoirs in Framingham, and a conduit from that town to Chestnut-hill reservoir in Brookline. In 1881 Sudbury River furnished to Boston more than twice the quantity of water supplied from Lake Cochituate; and steps have already been taken to increase still further the storage-capacity of the system. The volume begins with a discussion of the sources of supply, the rainfall, and the storage-capacity of the reservoirs. Next follows a general description of the dams and reservoirs, and of the several sections of the work, in all its engineering features. The quality of the water, the gauging of the river, and a discussion of the capacity of the conduit, and the flow of water over weirs, conclude the body of the work. The appendix contains valuable tables on water supply hydraulics, and a large amount of information for the practising engineer. The work is illustrated with 69 large plates, commencing with a map of the Sudbury River watershed, and giving very fully the constructive details of the dams and conduits. To give the city 40,000,000 gallons of water daily, it is estimated that the storage-reservoirs on Sudbury River should have a capacity of 4,900,000,000 gallons. So far, three reservoirs only have been built; having a capacity, with that of Farm Pond, of 2,000,000,000 gallons, intended to give a supply of 20,000,000 gallons daily to the city.

THE PROGRESS OF THE ARLBERG TUNNEL.—Some interesting particulars are given concerning the Arlberg tunnel, the boring of which is being pushed with great energy and success. The present road over the Arlberg, which forms the frontier between Austria and Switzerland, is 5,400 feet above the level of the sea; but the tunnel is much lower down, the opening on the Tyrol side being 4,030 feet, and that on the Swiss side 3,770 feet above sea level Its total length will be 10,270 meters (11,161 yards, or six miles and 601 yards), and it runs for the most part through a formation of mica schist. The method of excavation differs from that practised in the making of the St. Gothard tunnel. Instead of piercing the upper part of the passage first, and working down, the Austrian Engineers have preferred to begin at the base and work upwards. The face of the rock is drilled by perforators actuated by compressed air, which is pumped into the tunnel by turbines stationed at its two extremities. When a sufficient number of holes have been drilled they are charged with dynamite and exploded After the blast the débris is removed by trucks, which follow closely on the track of the perforators, and a few minutes later the drilling is going on as rapidly as before. The drift thus made is 2.75 meters wide and 2.50 meters high. While one drift is being driven below, another, to which access is gained by vertical shafts, is being driven above. This work has necessarily to be done by hand, and the rubbish is shunted through openings, made for the purpose, into an inferior gallery. Until very lately the ventilation has given rise to no difficulty, and the heat has rarely exceeded 14 deg. Cent. (58 deg. Fnhr.). The contractors have undertaken to make an average advance of 6.60 meters a day. For every day they exceed the given time they will be mulct in a penalty of £68; for every day gained they will receive a premium of £68. So far the contractors have kept well up to time. On not a few occasions the agreed rate of advance has been more than doubled. From January, 1881, when the work began, to September 30th, 1882, the length pierced on the east side was 2,976 meters, on the west 2,643, together 5,619, equal to 8.80 meters daily, figures which are highly significant of the prog ress made of late years in the method of boring great tunnels. In the month of February last the rate of advance per day was 4.68 meters on the east side, 4.74 on the west side, and but for the scarcity of water, owing to the freezing of the sources of supply, a still better average would have been made. On the west side there is now a stretch of 3,070 meters practicable for locomotion, while on the east side the completed stretch is only 1,430 meters. Up to the end of February the quantity of earth and rock removed amounted to 429.082 cubic meters, and the walling to that date executed measured 121.511 cubic meters. The tunnel is expected to be completed and the line ready for opening by the autumn of 1884.—*Iron.*

THE boring of the Alberg tunnel, which recently suffered some interruption by reason of the nature of the material encountered, is now progressing at the rate of ten meters a day. At the end of last year there remained no more than 3468 meters to pierce.

RAILWAY NOTES.

RAILWAY ACCIDENTS IN 1882.—The blue-book just issued gives returns of accidents and casualties as reported to the Board of Trade by the several railway companies in the United Kingdom during 1882. From this it appears that the total number killed in 1882 was 1,121, as against 1,096 in 1881, and of injured 4,601, as against 4,571 in 1881. The passengers killed were 127, and injured, 1,739, as against 108 killed and 1,860 injured in the previous year.

A SWISS ELECTRIC RAILWAY.—It is proposed to build an electric railway between Saint-Moritz-les-Bains and Portresina, in Switzerland. The length will be 7,200 meters (4½ miles). Previous to the opening of the St. Gothard Tunnel the traffic between Switzerland and Italy passed by Coire and Chiavenna, across the Col du Jubier or the Col du Splugen. The railway on the Swiss side terminates at Coire, and during the next year the Italian system will be completed as far as Chiavenna. The projected electric railway will serve to connect Coire to Chiavenna by two routes. The distance from Coire to St. Moritz is 76.5 kilometers, and from St. Moritz to Chiavenna 48.8 kilometers, or 125 kilometers in all. It is only proposed to erect a short portion at first, and if successful to extend it, as there is ample water-power in the district.

NEW ALGERIAN RAILWAY.—A bill has been laid before the French Chamber of Deputies, the object of which is (1) to declare "of public utility" the proposed railway from Ménerville to Tizi-Ouzon, and (2) to approve a convention agreed upon between the Minister of Public Works and the Eastern-Algerian Company. The concession for the Ménerville and Tizi-Ouzon line has been granted to this company. It is of a total length of about 31 miles, and connects the richest and most populous part of Kabylia with Algeria. It also establishes strategic communication between the upper Sebaon and Fort National on the one hand, and the capital of Algeria on the other. The scheme has been approved by the Minister of War, and the General Council of Bridges and Highways. By the convention, the company is bound to execute the work in four years, and the State guarantees interest at five per cent. on a capital of 676,694*l.* The company is permitted to issue stock to the value of 1,000,000*l.* It deposits 2,000*l.* caution money.

RHINELAND STEAM TRAMWAYS.—This line has been working since June, 1881, with five of Messrs. Merryweather's steam tramway locomotives. It is 5½ miles in length, and consists of a single line with three crossing stations. The roadway is over sand, and it is satisfactory to learn that the wearing parts of the engines give very little indication of want of repair. The traffic is very heavy in the summer time, owing to the many seaside excursions to Kat-

wyk. The metals weigh 15½ kilogrammes per meter, and are of steel with cross sleepers; and the cost of making the line was under £2,500 per mile, inclusive of the construction of three bridges. In summer the load behind each engine is not less than 20 tons. There is a driver only to each engine; the fire is filled up with coke at starting on each journey, and requires no attention *en route*; in fact, it is found in practice that it is unnecessary to open the furnace door at all *en route*. This is owing to the large capacity of the boiler and its regularity in steaming, it being of the horizontal type. The average consumption of coke is 10 lbs. to 12 lbs. per mile run. The coke is make from German coal, and is of a quality equal only to ordinary gas coke; otherwise the consumption would be even less. The cost of working, including all shop charges, is 2¼*d*. per mile, or the same as the Stockton line, which runs the same type of engine. The depot is well and economically managed; it is a plain shed with duplicate line of rails, with a pit to take three engines. The machinery consists of a half-horse power hot-air engine, which drives a pump for filling water tank, also a lathe with thirteen centres and a drilling machine; in addition, there is a smith's forge and two pairs of vises. This is the whole plant, with the exception of small hand tools The shop staff consists of one foreman at 30*s*. per week, and three cleaners at 20*s*. per week. The director states that he finds it much cheaper to get his duplicate parts from the constructors of the engines than to attempt to do the work at home, and this he appears to have proved through two years of successful running. In order to show the sharpness of the sand and the wear of tires, it is stated that a tire runs three months, then is turned up twice, and the average length of a tire is ten months.

IRON AND STEEL NOTES.

As in other departments, so in that of metallurgical science, the past year yields up no event of surpassing importance to place on record. One useful, practical improvement, however, there certainly is for notice, and that is the soaking-pit process of Mr. John Gjers, of Middlesbrough, which was brought under the notice of the members of the Iron and Steel Institute at Vienna last autumn. This process renders it now easy to roll a bloom into a rail or other finished article with its own initial heat, instead of having to submit the ingot to the heating furnace. This is effected by placing the ingots in pits, where, little or no heat being able to escape to the surface, and the ingots being surrounded by walls as hot as themselves, the surface heat of each ingot is greatly increased, and in about half an hour it is fit for the rolling mill. The process was introduced early last year at the Darlington Steel and Iron Company's works in Darlington and at the West Cumberland Steel Works, and at the close of the year at each place over 30,000 tons of ingots had been successfully treated. At Darlington they are now turning out a greatly-increased output by using the pits, and so much heat do the pits retain that, although work is stopped at noon on the Saturday, they are found to be sufficiently hot on the Monday morning, or even later on, to treat the first blow. Of the 30,000 tons treated at Darlington so far, about 10,000 tons have been soft steel for wire billets, and it is considered that the steel is actually improved for all purposes by the soaking pit process. That the invention is a thorough practical success has been established beyond all question, and there are several other works in England at which preparations are being made for its adoption. There is a prevailing opinion amongst railmakers that rails made from soaked ingots are better than if made from ingots put in the heating furnaces, for the danger of burning is entirely obviated. Another departure in metallurgical practice came under our notice in the early part of last year, at which time we gave particulars of it. This was the process of manufacturing iron direct from the ore by the aid of crude petroleum as fuel in a special revolving furnace, the invention of Dr. G. Durgee, of New York. There are two furnaces, an upper and a lower one, placed at slightly different levels, their total length being 120 feet, and their working capacity 100 tons of ore in 24 hours. The ores are pulverized, and then submitted to the action of an oxy-hydrogen flame produced by a blast of air with petroleum and coal-dust, thus to some extent following in wake of the Crampton coal-dust furnace. At the date mentioned some very successful and encouraging experiments were reported, but we have not heard a word nor seen a line about it since. From the Continent we recently received particulars of a new metal—"steel-iron." This metal is produced by pouring molten steel and molten iron into a cast-iron mould divided centrally by a thin sheet of iron, the fluid iron and steel being poured one on either side of the plate, which serves as a medium, welding both parts, steel and iron, completely together. By this process—which is stated to be the invention of Professor Keil—steel-iron of varying character is produced in various ways, and is said to meet the requirements of every class of manufacture, the new metal possessing the characteristics of both steel and iron. In May last year some stir was caused by the announcement of a new system producing of iron and steel, known as Bull's process. The chief features of the process were stated to be the calcination and heating of the charge, the introduction of highly-heated gas into the blast furnace at a certain stage, and the gradual removal of the fuel, the charge being maintained at certain stated levels according as it was desired to vary the percentage of carbon in the metal. Upon instituting inquiries at the time, we could not find that it had passed the experimental stage, but we have just heard that it is now intended to lay down plant and machinery in Sweden for testing it on a working scale.

In spite, however, of the various attempts to introduce new processes for producing steel, that of Sir Henry Bessemer still holds its own, whilst the adoptions of the basic or Thomas-Gilchrist process continue to increase. Turning to the broad question of the produce of steel throughout the world, from some recently

published statistics, it appears that there are now in England 23 steel works, with about 115 converters of a productive capacity of 1,461,000 tons per annum. Austria has 14 steel works, with 36 converters, and a capacity of 350,000 tons; Belgium, 4 steel works, with 18 converters, and a production of 380,000 tons; France, 7 works, with 34 converters, giving a production of 632,000 tons. Germany has 23 Bessemer and Thomas steel works, with 80 converters, and a productive capacity of about 1,3[illegible]0,000 tons; Russia has 5 works, with 10 converters, and a production of 100,000 tons; and Sweden, 35 converters of 80,000 tons capacity. In the United States there are in operation altogether 34 converters, with an annual production of 1,500,000 tons. The total number of converters in the world is therefore about 360, with an aggregate annual productive capacity in round numbers of 5,800,000 tons of steel. The substantial position occupied by the Thomas-Gilchrist process is well shown by the large amount of basic steel turned out during the month of October last. In this respect Germany holds the first position with an output of 25,170 tons of steel by eight firms. England stands next with an output of 9,500 tons by one firm. Austria shows an output of 7,700 tons by three firms; Belgium, 1,687 tons by one firm; Russia, 1,270 tons by one firm; and France, 1,240 tons by one firm. We thus have total output of basic steel for the month of October of 46,437 tons, by fifteen firms. When we consider that the process is still but little more than in its infancy, this output may be taken as a satisfactory proof that it is paying, and it is only reasonable to expect in so young a process that with increased practice the anticipations that the present cost will be reduced will be realized.—*Iron.*

BOOK NOTICES.

PUBLICATIONS RECEIVED.

ANNUAL REPORT OF THE ACUSHNET WATER BOARD (NEW BEDFORD.)

MONTHLY WEATHER REVIEW FOR MARCH. Washington: Gov't Printing Office.

BULLETIN OF THE PHILOSOPHICAL SOCIETY OF WASHINGTON. Vols. 4 and 5.

REGULATIONS FOR LIGHTING BRIDGES OVER NAVIGABLE RIVERS. Washington: Light-House Establishment.

CHEMISTRY: INORGANIC AND ORGANIC. By Charles Loudon Bloxam. Fifth Edition. Philadelphia: P. Blakiston & Son.

No work on general chemistry enjoys a higher reputation than Bloxam's. Treating of both organic and inorganic compounds in a single volume it is one of the most convenient books of reference.

The determination to keep up with the progress of the science is sufficiently indicated by the numerous revisions to which the work is subjected.

The date of the completion of this edition is March, 1883. Price, $4.00.

DYNAMO-ELECTRIC MACHINERY. By Silvanus Thompson, M. S. T. E. Science Series No. 66. New York: D. Van Nostrand.

While the dynamo-electric Machine has, within a comparatively short period, achieved a most wonderful development in connection with the rapidly increasing application of electricity to the useful arts, yet the essential principles of its construction and operation appear to be but imperfectly understood, even by otherwise well-informed electricians. The forms in which it has appeared have already become so numerous and varied, and the conditions of its construction and use are so complex in their mutual relations, as not only to defy all general methods of mathematical investigation, but to a considerable extent even satisfactory classification.

The great value of Professor Thompson's work, especially to the student and investigator, is mainly due to his masterly power of analysis, as shown in his method of logically grouping the various dynamos in connection with certain typical forms, each possessing characteristic individual elements which are readily comprehended, and when once fixed in the mind, are instantly recognized wherever found. Hence, any existing machine, or any new machine which may hereafter appear, may be at once referred to its proper type, and the essential features of its organization compared with those of its rivals or predecessors.

The present series of lectures, like all the productions of its eminent author, are characterized not only by a thorough mastery of the subject, both in its theoretical and practical aspects, but by a felicity of classification, and a conciseness and clearness of statement and illustration, which literally leaves nothing to be desired. It forms a work which will be found alike indispensable to the educated electrician, the student, the inventor, and the intelligent mechanic or apprentice.

THE PORTER-ALLEN STEAM ENGINE. By Chas. T. Porter. Philadelphia: Southwark Foundry and Machine Co.

This book is simply descriptive of those parts of the Porter-Allen engine which are characteristic, but these are presented with a clearness of type and diagram that are rarely exhibited in treating of mechanical details.

THE STUDENT'S MECHANICS. By Walter R. Browne, M. A. London: Charles Griffin & Co.

This work differs from the ordinary elementary treatise on mechanics in the exceptional fulness with which the subjects of force and motion are presented.

Definitions and laws are expressed with great care and abundantly illustrated by examples.

The book will prove valuable to students who have completed a rudimentary course in mechanics and have begun the calculus. Price, $1.80.

FRENCH FOREST ORDINANCE OF 1669. Translated by John Croumbie Brown, LL.D. London: Simpkin, Marshall & Co.

Dr. Brown is an indefatigable champion of the conservancy of forests and contributes

more than any other English writer to the cause of forest culture and preservation.

Advocates of the cause in this country will find valuable counsel in the works of this author and will read with interest this sketch of laws of two centuries ago.

REPORT OF THE CHIEF OF ORDNANCE FOR THE YEAR 1882. Washington: Gov't Printing Office.

This report, aside from the uninteresting details which only government officials can regard with any show of interest, contains much that will be read with interest by those who care for the science of gunnery and the theory of projectiles. The plates are numerous and better than usual.

A MANUAL OF PRACTICAL HYGIENE. Sixth Edition. By Edmund A. Parkes, M. D., F. R. S. Philadelphia: P. Blakiston & Son.

More than the usual number of topics is embraced in this voluminous treatise on Hygiene. Water, air, food and habitation are all fully considered.

Aside from its value as a general treatise it affords the best possible guide to sanitary inspectors by supplying suggestions in regard to the seat of troublesome causes of disease.

Altogether it is the most complete and authoritative compendium of sanitary science that we know of. Price, $6.00.

MODERN PERSPECTIVE. By Wm. R. Ware. Boston: James R. Osgood.

Readers of the *American Architect and Building News* know something of the value of Prof. Ware's treatise on Perspective. The present book contains the papers published in the above mentioned journal, together with a large amount of additional matter including many plates.

Nothing seems wanting to make a complete text-book for the student. The plates are large and well executed. The text is in a separate volume. Price, $5.00.

A MANUAL OF MARINE ENGINEERING. By A. E. Seaton. London: Charles Griffin & Co.

This work has been prepared to supply a need of a guide to the application of theoretical principles to the practical construction of marine machinery. It is designed to aid both the engineer and draughtsman in their respective labors.

The treatise is exhaustive: text covering 425 royal octavo pages and is well illustrated. Price, $6 00.

USEFUL RULES AND TABLES. By William John Macquorn Rankine. Sixth Edition. London: Charles Griffin & Co.

This new edition of a well known table-book is enriched by a collection of tables and formulæ relating to electrical engineering. This latter has been compiled by Andrew Jamieson, C. E., F. R. S. E.

Aside from this important addition, which covers about seventy pages, the book is like the previous edition. Price $4 00.

MISCELLANEOUS.

A CHEAP brilliant black can, it is said, be produced on iron and steel by applying with a fine hair brush a mixture of turpentine and sulphur boiled together. We have not tried it.

IN water containing saltpetre and air free from carbonic acid, lead and zinc, in experiments made by Professor A. Wagner, were attacked most violently; tin and Britannia metal a little; copper, brass and German silver not at all. With air and carbonic acid present, zinc and lead were attacked most; copper, German silver and brass were not more acted upon than by distilled water; tin and Britannia metal were acted upon somewhat. None of the metals were dissolved when carbonic acid was absent; when it was present, perceptible quantities were dissolved. In carbonate of soda, and air free from carbonic acid, lead, copper, brass and German silver lost nothing in weight, but zinc, tin and Britannia metal were sensibly affected. Perceptible quantities of tin and Britannia metal were dissolved; none of the other metals entered into solution. It was not possible to pass carbonic acid into the solution, as this would convert the carbonate of soda into bicarbonate of soda.

A TRIAL of an electro-magnetic motor, aërial screw, and bichromate elements constructed by MM. Tissandier for their directing balloon, took place in their aëronautical workshop at Point du Tour, on January 26th, before a large number of electricians and aëronaunts. It was shown that the twenty-four elements, each of which weighs about six kilogrammes, give during almost three hours a current which rotates a screw of 2.85m. diameter, and about five meters of path, with a velocity of 150 turns in a minute. *Nature* says the motive power really developed may be estimated at that of four horses per hour. But in a paper read on the 22d ult. before the Academié des Sciéncés, M. Tissandier said: "The system, with a total weight equal to three men, gives during three hours the work of twelve to fifteen men. The two-vaned propeller—of steel wire and varnished silk—is driven by a small Siemens dynamo—120 turns of the former to 1200 of the latter; the battery being of thirty-four elements mounted in tension, and divided into four series. An element consists of a vulcanite box—four liters capacity—holding ten zinc and eleven carbon plates. Strong bichromate solution is let in or drawn off by raising or lowering a separate vessel connected by a tube with the battery." The weight of all the machinery and elements is a little less than 250 kilogrammes. The real effect on the air can only be found by experiments in the air, but according to measurements taken with a dynamometer of the horizontal tendency to motion, it is about the same as in the experiment tried by Dupuy de Lome. The motive power of Dupuy de Lome having been obtained with eight men working his large screw, whose diameter was nine meters, it may be inferred that the results in the present case will be more advantageous in the

ratio of two and one-half to one. These results are not very powerful when compared with the immense power of aërial currents. But MM. Tissandier have no intention of directing their balloon against st ong winds. Their object is to organize an apparatus with which rational experiments may be made in the air, and they have taken advantage of the most recent improvements of science. If their elongated balloon answer their wishes, a real advance will be registered in the history of aëronautics.

In an article on "Freeboard," from a seaman's point of view, the *Nautical Magazine* says: "Superstructures, no matter how or in what part of the ship erected, cannot add to the safety of the platform as represented by the lowest portion of main-deck, and consequently any reduction on that score is manifestly a reduction of safety. The matter resolves itself into a simple argument. In estimating the freeboard for a flush-decked vessel, the chief desideratum is to have such an amount of clear side as will afford a safe platform on which the crew can go to and fro to work the ship, &c. Now, as vessels are constructed at present, is it not a fact that in loading and putting a ship down in the water, the deck ceases to be a safe platform for human life long before there is any danger of the deck, bulkheads or hatches being stove in? Clearly, then, safety of platform is the point at which we must stop, and the erection of no superstructure either before or abaft the lowest part of that platform can make it any safer."

Mr. J. P. Maginnis, of 10 Victoria-chambers, Westminster, has made two new drawing instruments, which have wide ranges of application, and will no doubt be extensively used. One is what he calls a dead-beat adjustable sectioner, an instrument by the aid of which parallel lines may be ruled at any distance apart, from about 6 or 8 to 200 to the inch for sectioning, shading and cross hatching The other instrument is a universal sector, which is an ingenious though simple in-trument. It was specially designed for constructing scales to suit shrunk drawings and lithographs, but it is equally well adapted: (1) To divide a line of any length, up to 5in., into any number of equal parts; (2) to give the proportion, in decimals, of the distance between any two given points, to the distance between any other two given points; (3) to set off or determine angles; (4) to bisect, trisect, or otherwise sub-divide angles; (5) to find the center of a given circle; (6) to reduce or enlarge ordinates of railway or other sections, curves, &c., in any desired proportion.

Some time ago Mr. Herbert McLeod pointed out that the deterioration of ebonite surfaces was due to the combined action of light and air. He now writes to *Nature*: "Some time ago it was remarked to me that our laboratory—an old greenhouse—was too light, and as a result, all our india rubber tubes would rapidly deteriorate. This led me to submit some pieces of ordinary black india-rubber to the same treatment as the ebonite in the former experiments. On October 11, 1879, four pieces of caoutchouc connector of 5 mm. internal diameter were taken, two were placed in test tubes plugged with cotton wool, and the remaining two inclo ed in hermetically sealed tubes. One of the sealed tubes, and one of those plugged with cotton wool, were placed in a dark drawer, and the other pair in the laboratory window, with a north aspect, and in such a position that they were not under the influence of direct sunlight in the summer. To-day the specimens were examined. Both the sealed tubes were found to be slightly moist inside, and on opening them an organic odor, like that of an india-rubber shop, was perceived. The caoutchouc which had been exposed to air and light, was covered with a thin brown coating, and on being bent this coating cracked; the end which had been most exposed to the light was rather brittle, and could not be stretched without splitting. The other three specimens were unaltered. All four specimens were slightly acid to test paper, but the quantity of acid was too small to be determined. Mareck (*Chem. News*, xlvii. 25, from *Zeitschr. für Anal. Chem.* xxi.) has lately recommended the preservation of caoutchouc tubes, by keeping them in water when not in use. This is, no doubt, efficacious, in consequence of the exclusion of air."

The results of a fourth year's observations of periodic movements of the ground as indicated by spirit levels at Secheron, are given by M. Ph. Plantamour in the *Archives des Sciences*, of December 15th. The curves obtained from the east-west spirit level for the four years are strikingly similar in the manner in which they follow the thermal oscillations of the air. Different years show a notable difference in the epoch of maximum descent of the east side relatively to the minimum of mean temperature, and maximum rise of the same side relatively to the minimum of temperature. One is led, says *Nature*, to consider the maximum and minimum of temperature rather as accidents as regards the epoch at which they occur, and to attribute a preponderant influence to the distribution of mean temperatures during the four months November-February, and the four June-September. Probably, too, the degree of moisture influences largely the rapidity with which the deeper ground layers are affected by exterior temperature. The curve for the north-south level is also very similar to the previous ones: but has this peculiarity, that while the south side follow, in general, from October 1st to the end of September, the oscillations of external temperature—descending in winter and rising in summer—the intermediate variations of temperature have no inverse effect. The cause is at present unknown. Colonel van Orff's observations at Bogenhausen reveal oscillations of the ground similar to those at Secheron, only with greater amplitude south-north, and less east-west. M. Plantamour regrets that, excepting Colonel van Orff and M. d'Abbedie, no one, so far as he knows, has undertaken observations of the kind at any other station. They are easily made, and should yield important results.